University of Illinois Spring 2016

ECE 313: Hour Exam I

Wednesday, March 2, 2016
7:00 p.m. — 8:15 p.m.
Aa-Hs in DCL 1320,
Ht-Lf in ECEB 1013,
Lg-Np in ECEB 1015,
Nq-Zz in ECEB 1002

Name: (in BLOCK CAPITALS) ____________________

NetID: ____________________

Signature: ____________________

Section:
☐ E, 9:00 a.m. ☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheets of notes is permitted: both sides may be used. No electronic equipment (cell phones, etc.) allowed.

The exam consists of 6 problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 18 points ____________
2. 17 points ____________
3. 21 points ____________
4. 16 points ____________
5. 16 points ____________
6. 12 points ____________

Total (100 points) ____________

1. **[18 points]** Consider three events, A, B, C in a probability space. Let $P(AB) = 0.25$, $P(AB^c) = 0.25$. It is known that A and B are independent and that B^c and C are mutually exclusive. Find numerical values for the quantities below, and show your work.

   (a) Obtain $P(B)$.

   (b) Obtain $P(A^c B^c C)$.

   (c) Obtain $P(A^c B^c C^c)$.

2. **[17 points]** Temperature X in degrees F corresponds to temperature $(X - 32) * (5/9)$ in degrees C. For example, 50 °F is equivalent to 10 °C. According to www.climatestations.com, the standard deviation of the daily temperature (in Chicago, years 1872-2008) is 6 °F in July and 12 °F in January.

   (a) What is the standard deviation of the daily temperature in July, measured in °C? Give the numerical value and show your work.

   (b) What is the *variance* of the daily temperature in July, if temperature is measured in °F. Give both the numerical value and the units of the variance.

   (c) What is the ratio of the variance of the daily temperature in January to the variance of the daily temperature in July, both measured in °C. Give the numerical answer and briefly explain.

3. **[21 points]** Suppose two fair dice are rolled. Define the following three events:

- D is "doubles" (the two numbers rolled are equal)
- S is "the sum is seven"
- A is "the product is even."

(a) Which pair of events (from among D, S and A) are mutually exclusive?

(b) Find $P(D|A)$. Show your work!

(c) Find $P(A|S)$. Show your work!

4. **[16 points]** Suppose X is Binomial(n, p) under hypothesis H_0 and Binomial$(n, 1-p)$ under hypothesis H_1, such that $p < 0.5$ and n is odd.

   (a) Given $X = k$ for some fixed number k, derive the ML rule to decide on the underlying hypothesis. *Hint*: your decision will depend on k.

   (b) What is the probability of missed detection for $n = 5$ and $p = \frac{1}{3}$?

5. **[16 points]** There are two drawers. The left drawer contains two green socks, three red socks and five orange socks. The right drawer contains two red socks and two orange socks. A sock is randomly picked from the left drawer and put into the right drawer. Then a sock is randomly picked from the right drawer.

   (a) What is the probability that the sock picked from the right drawer is red?

   (b) What is the probability that the sock picked from the left drawer (and transferred to the right drawer) is red given that the sock picked from the right drawer is red?

6. [**12 points**] A bag contains $n \geq 11$ balls labelled 1 through n.

(a) Balls are randomly taken out one at a time.
Obtain $P\{\text{ball number 11 is the } 10^{th} \text{ ball to come out}\}$.

(b) Suppose that the n balls are randomly assigned to n people that are lined up. The first person in line compares his/her ball with the second person and the one with the smallest numbered ball wins. The winner then compares his/her ball with the third person in line and the one with the smallest numbered ball wins. And so on. Obtain $P\{\text{first person wins exactly ten rounds}\}$.