University of Illinois Spring 2015

ECE 313: Hour Exam II

Wednesday, April 15, 2015
7:00 p.m. — 8:15 p.m.

Name: (in BLOCK CAPITALS) _____

NetID: _____

Signature: _____

Section:
☐ E, 9:00 a.m. ☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ F, 1:00 p.m. ☐ B, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. No electronic equipment (cell phones, etc.) allowed.

The exam consists of 6 problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading	
1. 14 points	_____
2. 18 points	_____
3. 20 points	_____
4. 16 points	_____
5. 14 points	_____
6. 18 points	_____
Total (100 points)	_____

1. **[14 points]** Let c be a constant and X be a random variable with pdf

$$f_X(u) = \begin{cases} \frac{1}{9}u^2 & u \in [0, c], \\ 0 & \text{else.} \end{cases} .$$

(a) Obtain the value of c in order for $f_X(u)$ to be a valid pdf.

(b) Determine the CDF $F_X(u)$ for all u.

2. **[18 points]** Planes arrive at O'hare as a Poisson process at the rate of 50 planes per hour. Let N be the number of planes that arrive in the time period between 9am and 10am.

   (a) Calculate $E[N]$.

   (b) Calculate $E[N]$ conditioned on the fact that 120 planes arrive in the time period between 9am and 11am.

   (c) Calculate the probability that exactly 3 planes arrive in the minute between 9am and 9:01am, conditioned on the fact that 120 planes arrive in the time period between 9am and 11am. *Hint*: Use the Poisson approximation.

3. **[20 points]** Let X and Y be two random variables with joint pdf

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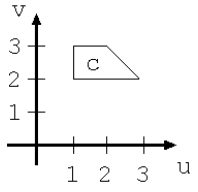
$$f_{X,Y}(u,v) = \begin{cases} c & 1 \le u \le 2,\ 2 \le v \le 3, \\ c & 2 \le u \le 3,\ 2 \le v \le 5-u, \\ 0 & \text{else.} \end{cases} ,$$



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where c is a constant.

(a) Determine the value of the constant c for $f_{X,Y}$ to be a valid joint pdf.

(b) Determine the marginal pdf of Y, $f_Y(v)$, for all v.

(c) Determine the conditional pdf $f_{X|Y}(u|v)$ for all u, v.

4. **[16 points]** The two parts of this problem are unrelated.

   (a) Let X be uniformly distributed in $[0, 1]$. Find the CDF for

   $$Y = 2|X - 1/2|.$$

   (b) Let Z be a random variable with pdf

   $$f_Z(u) = \begin{cases} 0 & u < 2, \\ \frac{1}{2} & u \in [2, 4], \\ 0 & u > 4. \end{cases},$$

   Determine a function $q(Z)$ such that the random variable $W = q(Z)$ is uniformly distributed on $[0, 1]$.

5. **[14 points]** The two parts of this problem are unrelated.

(a) Consider the following binary hypothesis testing problem. If hypothesis H_0 is true, the continuous random variable X is uniformly distributed on $(-1, 1)$, while if hypothesis H_1 is true, the pdf of X is $f_1(u) = \begin{cases} 1 - |u|, & |u| < 1, \\ 0, & \text{otherwise.} \end{cases}$
Find the decision region Γ_0 for the MAP (*maximum a posteriori probability*) decision rule if $\pi_0 = 1/4$. Remember that Γ_0 is the set of all real numbers such that if $X \in \Gamma_0$, the decision is that H_0 is the true hypothesis.

(b) Let X be a Gaussian random variable with mean μ and variance 4. Given that it is known that $P(X \geq 6) \approx 0.0228$, find the value of μ.

6. **[18 points]** Let X and Y be two random variables with joint pdf

$$f_{X,Y}(u,v) = \begin{cases} e^{-(u+v)} & u \geq 0,\ v \geq 0, \\ 0 & \text{else.} \end{cases}$$

(a) Are X and Y independent? Indicate why or why not.

(b) Let $Z = X + Y$. Find the CDF of Z.

(c) Find $P\{X > Y\}$.

Table 6.2: Q function, the area under the standard normal pdf to the right of x.

x	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

x	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**
0.0	0.5000000	0.4207403	0.3445783	0.2742531	0.2118553
1.0	0.1586553	0.1150697	0.0807567	0.0547993	0.0359303
2.0	0.0227501	0.0139034	0.0081975	0.0046612	0.0025552
3.0	0.0013500	0.0006872	0.0003370	0.0001591	0.0000724
4.0	0.0000317	0.0000134	0.0000054	0.0000021	0.0000008