

ECE 313: Problem Set 2

Discrete random variables

Due: Wednesday January 30, at 4 p.m..

Reading: ECE 313 notes, Chapter 1 and Sections 2.1-2.2.

1. **[Maximum and minimum values for probabilities]**

A , B and C are events with probabilities 0.6, 0.2, and 0.7 respectively.

- (a) What are the largest possible values of $P(A \cup B)$ and $P(A \cup C)$?
- (b) What are the smallest possible values of $P(A \cup B)$ and $P(A \cup C)$?
- (c) What are the largest possible values of $P(AB)$ and $P(AC)$?
- (d) What are the smallest possible values of $P(AB)$ and $P(AC)$?

2. **[Mean and standard deviation]**

Suppose three fair dice are rolled independently, so the sample space is $\Omega = \{(i, j, k) : 1 \leq i, j, k \leq 6\}$ and all outcomes are equally likely. Let X be the number showing on the first die, $X(i, j, k) = i$, and let Y be the random variable defined by $Y(i, j, k) = \min\{i, j, k\}$.

- (a) Derive the pmf of X and sketch it.
- (b) Find the mean $E[X]$ and standard deviation, σ_X , of X . Correct numerical answers are fine, but show your work.
- (c) Derive the pmf of Y and sketch it.
- (d) Find the mean $E[Y]$ and standard deviation, σ_Y , of Y .
- (e) Which is larger, σ_X or σ_Y ? Is that consistent with your sketches of the pmfs?

3. **[Die Roll]**

A fair die is rolled once. Let $f_1 \in \{1, 2, 3, 4, 5, 6\}$ denote the outcome. The die is then rolled repeatedly till an outcome f_2 that is *different* from f_1 occurs.

- (a) Find the probability that f_1 is even.
- (b) Find the probability that both f_1 and f_2 are even.
- (c) Find the probability that $f_1 + f_2 \leq 7$.

4. **[Defining Events]**

Express each of the following events in terms of the events A , B , and C , and the operations of complementation, union, and intersection.

- (a) at least one of the events A, B, C occurs;
- (b) none of the events A, B, C occurs;
- (c) all three events A, B, C occur;
- (d) exactly one of the events A, B, C occurs;
- (e) at most one of the events A, B, C occurs;
- (f) events A and B occur, but not C ;
- (g) either event A occurs, or if not then B also does not occur.