

ECE 313: Problem Set 10

Jointly distributed random variables including independent random variables

Due:	Wednesday, April 4 at 4 p.m.
Reading:	<i>ECE 313 Notes</i> Sections 4.1 - 4.4
Reminder:	Exam II is on Tuesday, April 10, 7:00 p.m. – 8:15 p.m. Location: Sections E (meets 9 am) and C (meets 10 am) 100 Material Sciences and Engineering Sections D (meets 11am) and F (meets 1pm) 151 Everitt Lab One two-sided 8.5"×11" sheet of notes allowed, with font size no smaller than 10 pt or equivalent handwriting. Bring a picture ID. The exam will cover the reading assignments, lectures, and problems associated with problem sets 1-10, with emphasis on problem sets 6-10.

1. [Joint probability mass functions]

Consider a certain tech-loving community, where 30 percent of residents own one smartphone, 35 percent own two, 20 percent own three, and 15 percent own none. Suppose that each smartphone is equally likely to be an iPhone or an Android phone. Let I and A be the number of iPhones and Android phones, respectively, that a randomly chosen resident owns.

- Find the joint probability mass function of I and A .
- Find the marginal probability mass function of I .
- Find the marginal probability mass function of A .
- Find $P\{I \neq A\}$
- Find $P\{I = 1|A = 2\}$.

2. [Joint probability mass functions]

A transmitted message consisting of five packets is known to have been received with exactly two packets in error. Each packet will be checked, one at a time, until the two packets with errors are identified. Let N_1 denote the number of packets checked until the first packet with errors is identified, and let N_2 denote the number of additional packets checked until the second packet with errors is identified.

- Find the joint probability mass function of N_1 and N_2 .
- Find the marginal probability mass function of N_1 .
- Find the marginal probability mass function of N_2 .
- Are N_1 and N_2 independent? Why or why not?
- Find $P\{N_1 = N_2\}$
- Find $P\{N_1 \leq N_2\}$
- Find $P\{N_1 = 2|N_2 = 2\}$.

3. [Joint probability density functions]

The jointly continuous random variables X and Y have joint pdf

$$f_{X,Y}(u,v) = \begin{cases} c(u^2 - v^2)e^{-u} & 0 \leq u < \infty \text{ and } -u \leq v \leq u \\ 0 & \text{else} \end{cases}.$$

- (a) Find the value of the constant of c .
- (b) Find the marginal pdf of X .
- (c) Find the marginal pdf of Y .
- (d) Are X and Y independent? Why or why not?
- (e) Find $P\{X + Y \geq 0\}$.

4. **[Joint probability density functions]**

The jointly continuous random variables X and Y have joint pdf

$$f_{X,Y}(u,v) = \begin{cases} 1.5 & 0 \leq u < 1, 0 \leq v < 1, 0 \leq u+v < 1, \\ 0.5, & 0 \leq u < 1, 0 \leq v < 1, 1 \leq u+v < 2, \end{cases}$$

and zero elsewhere.

- (a) Find the marginal pdf of Y .
- (b) Find $P\{X + Y \leq 1/2\}$.
- (c) Find $P\{X^2 + Y^2 \geq 1\}$.

5. **[Joint probability density functions]**

Suppose the continuous-type random variables X and Y are independent, where X is uniformly distributed over $[0, 1]$ and Y is exponentially distributed with parameter $\lambda = 2$. Note that $E[X] = E[Y]$.

- (a) Find the marginal pdf of Y .
- (b) Find $P\{Y \geq X\}$