ECE 313: Problem Set 8

Scaling rule for pdfs, Gaussian distribution, central limit theorem, ML parameter estimation

Due: Wednesday, March 14 at 4 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.6–3.7.

1. **[Exam Grading]**
Suppose students in a class (not ECE313) take an exam. Assume the exam scores are distributed according to a Gaussian distribution with mean 73 and standard deviation 16. Letter grades are assigned to students according to one of the following grading policies.

Policy 1	
Scores	Grade
85 or more	A
65 or more	B
45 or more	C
Less than 45	D

Policy 2	
Ranking	Grade
Top 15%	A
Top 40%	B
Top 80 %	C
Not in top 80%	D

   (a) Using Tables 6.1 and 6.2 in the lecture notes, find the percentage of students expected to A, B, C, and D under grading policy 1.

   (b) Using the same tables, find the cut-off scores between A and B, B and C, and C and D under grading policy 2. It is not necessary that the cut-off scores must be integers.

2. **[Gaussian Noise]**
Suppose a wireless communication system consists of a transmitter and a receiver. If the transmitter emits a radio signal with transmission amplitude v, this signal is attenuated by the air and is received by the receiver together with noise, denoted by N. Let $Y = \alpha v + N$ be the received signal amplitude, where $\alpha = 0.01$ is the attenuation factor and N is a Gaussian random variable with mean $\mu = 0$ and variance $\sigma^2 = 4$.

   (a) Find the pdf $f_Y(u)$ of the received signal amplitude Y for $v = 10$.

   (b) (Only for this part) Suppose that σ is unknown, and v and α are known to be 10 and 0.01, respectively. If the received signal amplitude Y is 0.03, find the ML estimator $\hat{\sigma}_{ML}$ of σ.

   (c) Suppose the transmitter wants to deliver one bit to the receiver using this wireless communication system. If the bit is zero, the transmitter transmits with $v = -10$. If the bit is one, the transmitter transmits with $v = 10$. Sketch the distributions of the received signal amplitude Y conditioned on that the bit being zero and conditioned on the bit being one.

   (d) Suppose the receiver output is zero if $Y < 0$ and one if $Y \geq 0$. An error is said to occur if the receiver output is not equal to the transmitted bit. Find the probability of error, given the transmitted bit is one.

3. **[Random Walk]**
Bob and Alice are at the starting point to race toward a goal, which is $10,000$ meters away. A coin is repeatedly tossed $10,000$ times. Each time, if a head comes up, Bob advances one meter. If a tail comes up, Alice advances one meter.

   (a) Express the distance between Alice and Bob after the 10,000 coin flips in terms of X, where X is the number of times heads shows.

   (b) Using the central limit theorem, find the probability that the distance between Bob and Alice is greater than or equal to 300 meters. (To be definite, you do not need to consider the continuity correction.)

4. **[Evolution of Stock Price]**
Suppose you bought a stock with a current price Y_0. Let Y_t denote your stock price after t days from today ($t \geq 0$). Everyday, your stock price increases by the factor 1.01 with probability 0.7 or decreases by the factor $(1.01)^2$ with probability 0.3. In other words, the stock price after $(t+1)$ days is given by

$$Y_{t+1} = \begin{cases} (1.01)Y_t & \text{with probability 0.7,} \\ \dfrac{Y_t}{(1.01)^2} & \text{otherwise.} \end{cases}$$

   (a) Let X be the number of days out of the first 1000 days that the stock price increases. Express the stock price Y_{1000} after 1000 days in terms of X.

   (b) Find the mean of Y_{1000}. The formula for $(a+b)^n$ in appendix 6.2 might be helpful. Your answer should be a constant times Y_0.

   (c) Using the central limit theorem, find the probability that you earn 200% or higher profit from the stock, i.e., $P\{Y_{1000} \geq 3Y_0\}$. (To be definite, you do not need to consider the continuity correction.)