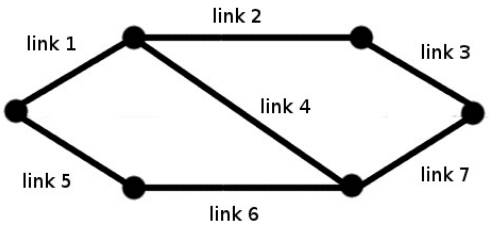
ECE 313: Problem Set 6
Reliability and CDFs

Due: Wednesday, February 29 at 4 p.m.
Reading: *313 Course Notes* Sections 2.12-3.1

1. **[Capacity of a flow network]**
Consider the following $s-t$ flow network, where link $i \in \{2, \ldots, 6\}$ fails with probability p_i, while links 1 and 7 never fail. Let c_i be the capacity (see Section 2.12.3) of link i, then $c_1 = 30$, $c_2 = 10$, $c_3 = 15$, $c_4 = 20$, $c_5 = 20$, $c_6 = 15$, $c_7 = 15$.



   (a) What values can the capacity of this network take?
   (b) Find the distribution (pmf) of its capacity.
   (c) Find the numerical values of the pmf of the capacity if $p_i = \frac{1}{i}$ for all $i \in \{2, \ldots, 6\}$.
   (d) Using the numerical values from part (c), find the expected capacity of the network.

2. **[Analysis of a three-dimensional array code]**
This problem extends the analysis of Section 2.12.4 to a three dimensional array code. Consider a three dimensional array code with $7^3 = 343$ data bits and a total of $8^3 = 512$ bits so that the bits in every eight bit line (parallel to x axis, y axis, or z axis) has even parity.

   (a) What is the minimum number of bit errors in a nonzero undetected error pattern? Describe an error pattern with that many bit errors.
   (b) Suppose each bit is in error with probability $p = 0.001$, independently of the other bits. Using a union bound based on your answer to part (a), find an upper bound on the probability of undetected errors. (Hint: See www.ohrt.com/odds/binomial.php for a binomial coefficient calculator.)
   (c) How many undetected error patters are there with the minimum nonzero number of bit errors?
   (d) Show that the next smallest number of errors possible in an undetected error pattern is larger by at least four.
   (e) Using a union bound and the answers to (c) and (d), find a tighter upper bound on the probability of undetected error. (Hint: See the last part of Section 2.12.4 of the notes for a similar analysis.)

3. **[CDFs]**
Consider the following function:

$$F(u) = \begin{cases} 0 & u < -5 \\ a & u = -5 \\ e^u & u \in (-5, -1) \\ b & u = -1 \\ \frac{1}{2} & u \in (-1, 1) \\ \frac{1}{8}u + \frac{3}{8} & u \in [1, 3) \\ 1 - ce^{-u} & u \geq 3 \end{cases}$$

(a) Obtain the values (or range of values) for a, b, and c such that $F(u)$ is a valid CDF.

(b) Sketch $F(u)$.

For the remainder of the problem supppose that $F(u)$ is the CDF of the random variable X, i.e. $F_X(u) = F(u)$.

(c) Find $P\{X = -5\}$.

(d) Find $P\{X = -2\}$.

(e) Find $P\{X \leq -5\}$.

(f) Find $P\{X \geq -1\}$.

(g) Find $P\{X = 0\}$.

(h) Find $P\{X = 3\}$.