University of Illinois **Spring 2012**

ECE 313: Problem Set 1
Axioms of probability and calculating the sizes of sets

Due: Wednesday, January 25 at 4 p.m.
Reading: *ECE 313 Course Notes,* Chapter 1

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Wednesdays, and is due by 4:00 p.m. on the following Wednesday. You must drop your homework into the ECE 313 lockbox (#20) in the Everitt Laboratory basement (east side) by this deadline. Late homeworks will not be accepted without prior permission from the instructor. All assignments must be submitted stapled if they consist of more than one sheet with the following heading in block capital letters with at least 12pt size font or equivalent neat handwriting, on the top right corner of the first page:
NAME AS IT APPEARS ON COMPASS
NETID
SECTION
PROBLEM SET #
The section should be one of E,C,D,F, and the problem set number an integer. Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. [**Defining a set of outcomes**]
   Ten balls, numbered one through ten, are initially in a bag. Three balls are drawn out, one at a time, without replacement.

   (a) Define a sample space Ω describing the possible outcomes of this experiment. To be definite, suppose the order the three balls are drawn out is important. Explain how the elements of your set correspond to outcomes of the experiment.

   (b) What is the cardinality of Ω?

2. [**Displaying outcomes in a two event Karnaugh map**]
   Two fair dice are rolled. Let A be the event the sum is even and B be the event at least one of the numbers rolled is three.

   (a) Display the outcomes in a Karnaugh map.

   (b) Determine $P(AB)$.

3. [**A three event Karnaugh puzzle**]
   Suppose A, B, and C are events such that: $P(A) = P(B) = P(C) = 0.3$, $P(AB) = 3P(ABC)$, $P(A \cup C) = P(B \cup C) = 0.5$, and $P(A^cB^cC^c) = 0.48$. Sketch a Karnaugh map showing the probabilities of $ABC, ABC^c, \ldots, A^cB^cC^c$. Show your work.

4. **[Selecting socks at random]**
Suppose there are eight socks in a bag, numbered one through eight, which can be grouped into four pairs: $\{1,2\}$, $\{3,4\}$, $\{5,6\}$, or $\{7,8\}$. The socks of each pair have the same color; different pairs have different colors. Suppose there are four (distinct!) people present, and one at a time, they each draw two socks out of the bag, without replacement. Suppose all socks feel the same, so when two socks are drawn from the bag, all possibilities have equal probability. Let M be the event that each person draws a matching pair of socks.

   (a) Define a sample space Ω for this experiment. Suppose that the order that the people draw the socks doesn't matter–all that is recorded is which two socks each person selects.

   (b) Determine $|\Omega|$, the cardinality of Ω.

   (c) Determine the number of outcomes in M.

   (d) Find $P(M)$.

   (e) Find a short way to calculate $P(M)$ that does't require finding $|M|$ and $|\Omega|$. (Hint: Write $P(M)$ as one over an integer and factor the integer.)

5. **[Two more poker hands]**
Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3, with all possibilities being equally likely.

   (a) *TWO PAIR* is the event that two cards both have one number, two other cards both have some other number, and the fifth card has a number different from the other two numbers. Find $P(\textit{TWO PAIR})$.

   (b) *THREE OF A KIND* is the event that three of the cards all have the same number, and the other cards have numbers different from each other and different from the three with the same number. Find $P(\textit{THREE OF A KIND})$.

6. **[Some identities satisfied by binomial coefficients]**
Using only the fact that $\binom{n}{k}$ is the number of ways to select a set of k objects from a set of n objects, explain in words why each of the following identities is true. The idea is to identify how to count something two different ways. For example, to explain why $2^n = \sum_{k=0}^{n} \binom{n}{k}$, you could note that 2^n is the total number of subsets of a set of n objects, because there are two choices for each object (i.e. include or not include in the set) and the n choices are independent. The right hand side is also the number of subsets of a set of n objects, with the k^{th} term being the number of such subsets of cardinality k.

   (a) $\binom{n}{k} = \binom{n}{n-k}$ for $0 \leq k \leq n$.

   (b) $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ for $1 \leq k \leq n$.

   (c) $\binom{n}{k} = \sum_{l=k}^{n} \binom{l-1}{k-1}$ for $1 \leq k \leq n$. (Hint: If a set of k objects is selected from among n objects numbered one through n, what are the possible values of the highest numbered object selected?)