University of Illinois Spring 2012

ECE 313: Hour Exam II

Tuesday April 10, 2012
7:00 p.m. — 8:15 p.m.
100 Material Sciences and Engineering Building & 151 Everitt Laboratory

1. [**10 points**] Suppose each corner of a three dimensional cube burns out with probability 0.001, independently of the other corners.
   (a) [**5 points**] Find a numerical upper bound, using the union bound, on the probability there exists two neighboring corners that both burn out. Explain your answer.
   (b) [**5 points**] Find a numerical upper bound, using the union bound, on the probability that two or more corners of the cube both burn out. It doesn't matter whether the burned out corners are neighbors. Explain your answer.

2. [**15 points**] Suppose disk crashes in a particular cloud computing center occur according to a Poisson process with mean rate $\lambda = 0.1$ per hour, 24 hours per day, seven days a week (i.e. 24/7). Find the *numerical values* of each of the following three quantities, and *explain or show your work.*
   (a) [**5 points**] The expected number of disk crashes in a 24 hour period.
   (b) ([**5 points**] The probability there are exactly three disk crashes in a five hour period. (The answer can include powers of e which do not need to be numerically calculated.)
   (c) [**5 points**] The mean number of hours from the beginning of a particular day, until three disks have crashed.

3. [**18 points**] Suppose you have three mutually independent random variables: $X \sim N(2,4)$, $Y \sim N(7,9)$, and $B \sim Bernoulli(2/3)$. Let $Z = XB + Y(1-B)$. (Note: Your answers can be expressed using the Q function or Φ function for the standard normal.)
   (a) [**5 points**] Find $P\{Z \geq 4 | B = 1\}$.
   (b) [**2 points**] Find $P\{Z = 4 | B = 1\}$.
   (c) [**5 points**] Find $P\{Z \leq 4 | B = 0\}$.
   (d) [**6 points**] Find $P\{Z \geq 4\}$.

4. [**7 points**] Let $X \sim Uniform\left[\frac{1}{a}, \frac{2}{a}\right]$. Suppose a is an unknown positive parameter and it is observed that $X = 3$. Find the maximum likelihood estimate $\hat{a}_{ML}$.

5. [**25 points**] Suppose X is an exponentially distributed random variable with parameter $\lambda > 1$,
$$f_X(u) = \begin{cases} \lambda e^{-\lambda u} & u \geq 0 \\ 0 & \text{else} \end{cases}$$
   and suppose $Y = e^X$.
   (a) [**8 points**] What is $E[Y]$? (Hint: Give a simple answer depending on λ.)
   (b) [**7 points**] What is the support of the pdf $f_Y(v)$ (the set of v for which $f_Y(v) \neq 0$)?
   (c) [**10 points**] For the set of v that you specified in part (b), find the CDF $F_Y(v)$.

6. [**25 points**] Suppose X and Y are jointly continuous-type random variables with joint pdf:
$$f_{X,Y}(u,v) = \begin{cases} \alpha(u+v^2) & \text{if } 0 \leq u \leq 1 \text{ and } 0 \leq v \leq 2u, \\ 0 & \text{otherwise,} \end{cases}$$
   where α is a positive real that you will find.

(a) **[6 points]** Determine whether X and Y are independent. Justify your answer.

(b) **[9 points]** Find the marginal pdf $f_X(u)$. (You do not need to find α for this part of the problem, but express your answer using α. Be sure to specify the pdf over the entire line, including where it is zero.)

(c) **[5 points]** Find α.

(d) **[5 points]** For what values of u is the conditional pdf $f_{Y|X}(v|u)$ well defined for all v?