University of Illinois Spring 2010

ECE 313: Problem Set 7

Decision Making; Independent Events; System Reliability

Due:	Wednesday March 10 at 4 p.m.
Reading:	Ross, Chapter 3; PowerPoint Lecture Slides 15-19
Noncredit Exercises:	Chapter 3: Problems 53, 58, 59, 62, 63, 70-74, 78, 81
	Theoretical Exercises 6, 7(a), 25, 26; Self-Test Problems 15-26.

This Problem Set contains seven problems.

1. [**Mutually exclusive events**] Let A and B denote two mutually exclusive events that can occur on a trial of an experiment. Repeated independent trials of the experiment are carried out until either the event A or the event B occurs. What is the probability that A occurs before B does? (See Example 4h in Chapter 3 of Ross)

2. [**Detection problem with geometric distribution vs. Poisson distribution**] Consider a detection problem with the following two hypotheses for an observation X:
H_0: X has the Poisson distribution with parameter $\lambda = 10$:
H_1: X has the geometric distribution with parameter p = 0:1:
   (a) To get some intuition about this problem, calculate the mean, variance, and standard deviation of X under H_0 and under H_1:
   (b) Describe the ML decision rule. Express it as directly in terms of X as possible.
   (c) Describe the MAP decision rule, under the assumption that H_0 is a priori 5 times more likely than H_1 (i.e, $\pi_0/\pi_1 = 5$): Express the rule as directly in terms of X as possible.

3. [**Independent events**] An experiment consists of three independent tosses of a coin. Let A and B respectively denote the events that the first and third tosses result in a Head, and let C denote the event that exactly two Heads occur and they occur on two successive tosses. Suppose that the coin is fair.
   (a) Are A and B physically independent events? Explain.
   (b) Are A and C stochastically independent events? Are they physically independent events? That is, are A and C independent if the coin were a biased coin?
   (c) Are B and C stochastically independent events? Are they physically independent events? That is, are B and C independent if the coin were a biased coin?
   (d) Are A, B, and C independent events?
   (e) Are A, B, and C pairwise independent events?

4. [**Fault tolerance**] The IlliniAuto Company needs to decide which of the following two methods provides more reliable transportation:

- a single gigantic car with N engines, N transmissions, N brakes, ... etc. that works (i.e. provides us with transportation) as long as at least one of its engines and at least one of its transmissions, and at least one of its brakes ... works.
- N separate ordinary cars that fail as soon as any one of their parts fail, but which together provide us with transportation as long as at least one car is in working condition.

Each car is made of M different types of parts, and (at least) one part of each different type must work for the car to work. Each part fails with probability p and all the failures are independent events.

(a) For each method, find the probability of system failure in terms of p, N and M.

(b) Suppose that M = 5 and p = 0.2. If it is desired that the system failure probability be less than 0.001, what should N be with each method?

(c) Repeat part (b) assuming that M = 1000.

5. [**Likely starry eyes**] A photodetector counts photons for 10 nanoseconds to decide if a distant light source is emitting light. When the light source is not emitting light, some photons are still counted by the detector due to the ambient background radiation. The number of photons counted in 10 nanoseconds is modeled as a Poisson random variable X whose parameter l has value ln(9) if the light source is not emitting light (hypothesis H_0), and value ln(27) if the source is emitting light (hypothesis H_1). The maximum-likelihood detector decides that H_1 is the true hypothesis if and only if the likelihood ratio $\Lambda(u) = p_1(u) / p_0(u)$ exceeds 1.

(a) Determine the value of $\Lambda(k)$ when k photons have been counted.

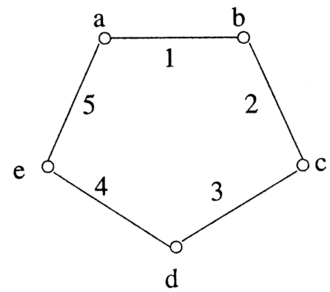
(b) Determine the value(s) of X that result in a decision in favor of hypothesis H_1.

(c) Compute the false alarm probability P_{FA} of the maximum-likelihood decision rule.

(d) Compute the missed detection or false dismissal probability P_{MD} of the maximum-likelihood decision rule.

6. [**A likely reliability challenge of multiple connections**] A certain Internet service provider in a midsize central Illinois city relies on k separate connections between the city and neighboring cities, to connect its customers to the Internet. Based on past experience, management assumes that a given connection will be down on a given day with probability p = 0.001, independently of what happens on other days or with other connections. Total outage is said to occur if all connections are down on the same day. How large should k be so that the probability total outage occurs at least one day in a year is less than or equal to 0.001?

7. [**Reliability of a self-healing ring communication network**] Consider the ring network shown, with nodes a, b, c, d, and e, and links numbered 1 through 5.



The links are full duplex, meaning they can be used in both directions simultaneously, but each link is assumed to fail in a given time period with probability $p = 0.001$. A pair of nodes is said to be connected if they are connected by at least one path with no failed links. The network is said to be connected if all pairs of nodes are connected. Find the probabilities of the following events:

(a) the network is not connected
(b) exactly two links fail
(c) nodes a and b are not connected
(d) nodes a and c are not connected