

ECE 313: Problem Set 1

Calculus Review

Due: Wednesday January 28 at 4 p.m..
Reading: *Ross* Chapter 1.1–1.4, Chapter 2.1–2.5.
Noncredit Exercises: Chap. 1: Probs. 1-5,7,9.
Theoretical exercises 4,8,13. Self-test probs. 1-15.
Chap. 2: Probs. 3,4,9,10, 11-14.
Theoretical exercises 1-3,6,7,10,11,12,16,19,20.
Self-test probs. 1-8.

About the credit problems (below): Calculus, a prerequisite of this course, will be used mainly in the second half of the semester. The parts of calculus primarily needed are integration and differentiation, Taylor series, l'Hospital's Rule, integration by parts, and double integrals (setting up limits of integration and change of variables such as in rectangular to polar coordinates). This problem set will help you review these topics, and identify areas in which you may need additional review.

1. [Geometric, MacLaurin and Taylor series]

Solve the following.

- (a) Prove that $1 + x + x^2 + \cdots + x^{n-1} = \frac{1 - x^n}{1 - x}$ for $x \neq 1$ and integer $n \geq 1$.
- (b) Assuming $|x| < 1$, find the sum $1 + x + x^2 + \cdots$.
Note, by definition this sum is the limit of the sum $1 + x + x^2 + \cdots + x^{n-1}$ as $n \rightarrow \infty$.
- (c) Use the chain rule to compute the k^{th} derivative of the function $f(x) = (1 + x)^n$ for $0 < k \leq n$.
- (d) Use your result from part (c) to compute the first $n + 1$ terms of the MacLaurin series for $(1 + x)^n$. Recall, the MacLaurin series is merely the Taylor series expansion of a function about the point $x = 0$.
- (e) Use the chain rule to compute the k^{th} derivative of the function $f(x) = (1 + x)^n$ for $k > n$, and give the complete MacLaurin series for $f(x) = (1 + x)^n$.

2. [Inverse function]

Let f be the function defined by $f(x) = \frac{2}{e^x + 1}$ for $-\infty < x < \infty$. Let g denote the inverse function of f , so that $g(f(x)) = x$ for all x . Find g . (Hint: The domain of g can be smaller than the whole real line. Draw a picture.)

3. [Extrema of functions]

Solve the following.

- (a) Find the maxima (plural) of $f(x) = x^4(1.001)^{-x^2}$ on the real line.
- (b) Does the function $f(x) = x^4(1.001)^{x^2}$ have a maximum value at some finite x ? Explain.

4. [Some definite integrals]

Find the values of the following **definite** integrals:

$$(a) \int_{-1}^1 |x| dx; \quad (b) \int_0^1 x(1-x^2)^{11} dx; \quad (c) \int_0^1 x^2 \exp(-x) dx;$$

$$(d) \int_{-\infty}^{\infty} x^3 \exp(-x^2/2) dx.$$

Hint: Plotting the integrand might give you some clues.

5. [Derivatives and integrals]

Let $\frac{d}{dx}f(x) = g(x)$, $-\infty < x < \infty$ and let C denote an arbitrary constant. Which of the following statements is true for all x ?

$$(a) \frac{d}{dx}f(-x) = -g(-x) \quad (b) \frac{d}{dx}f(x^2/2) = xg(x^2/2) \quad (c) \frac{d}{dx} \exp(f(x^2)) = g(x^2) \exp(f(x^2))$$

$$(d) \int g(-x) dx = f(-x) + C \quad (e) \int g(x^2/2) dx = \frac{f(x^2/2)}{x} + C \quad (f) \int \frac{g(x)}{f(x)} dx = \ln(f(x)) + C$$

6. [Double integrals]

Evaluate the following **definite** two-dimensional integrals over the specified domains of integration:

$$(a) \int \int f(x, y) = \max(x, y), \text{ over the region } \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

$$(b) \int \int f(x, y) = (x^2 + y^2)^{-4}, \text{ over the region } \{(x, y) : x^2 + y^2 > 1\}.$$