University of Illinois Spring 2009

ECE 313: Hour Exam II

Monday April 13, 2009
7:00 p.m. — 8:00 p.m.
100 Noyes Laboratory

1. **[20 points]** *Throughout this problem, A, B, and C denote events of probabilities 0.1, 0.2 and 0.2 respectively.* In different parts of this problem, you are asked to find the probabilities of some events under various assumptions about A, B, and C. *DO NOT carry over the assumptions in one part to another part of the problem.*

   (a) **[7 points]** Find $P(A^c \cup B^c \cup C^c)$ assuming that A, B, and C are *mutually independent events.*

   (b) **[6 points]** Find $P(A^c \cup B^c \cup C^c)$ assuming that A, B, and C are *mutually exclusive events.*

   (c) **[7 points]** Find the value of $P(A \cup BC)$ assuming that A and B are *mutually exclusive events* and B and C are *mutually independent events.*

2. **[30 points]** Consider the following binary hypothesis testing problem. If hypothesis H_0 is true, the continuous random variable $\mathbb{X}$ is uniformly distributed on the interval $(-2, 2)$, while if hypothesis H_1 is true, the pdf of $\mathbb{X}$ is $f_1(u) = \begin{cases} \frac{1}{4}(2-|u|), & |u| < 2, \\ 0, & \text{otherwise.} \end{cases}$

   (a) **[14 points]** The *maximum-likelihood* decision rule can be stated in the form $|\mathbb{X}| \underset{\mathsf{H}_{1-x}}{\overset{\mathsf{H}_x}{\gtrless}} \eta$. Specify whether x denotes 0 or 1, and find the values of η, the probability of false alarm P_{FA}, and the probability of missed detection P_{MD}.

   (b) **[4 points]** Suppose the hypotheses have *a priori* probabilities $\pi_0 = 1/3$ and $\pi_1 = 2/3$. What is the error probability $P(E)$ of the maximum-likelihood decision rule?

   (c) **[12 points]** The MAP (also known as the minimum-error-probability or Bayesian) decision rule can be stated in the form $|\mathbb{X}| \underset{\mathsf{H}_{1-x}}{\overset{\mathsf{H}_x}{\gtrless}} \xi$. Specify whether x denotes 0 or 1, and find the values of ξ and the error probability $P(E)$.

3. **[25 points]** For this problem, you will need to use the table of values for the unit Gaussian CDF $\Phi(x)$ on the last page of this exam booklet.

   The random variable X has pdf $f_X(u) = \begin{cases} \dfrac{K}{2\sqrt{2\pi}} \exp\left(-\dfrac{(u-2)^2}{8}\right), & 0 \le u \le 4, \\ 0, & \text{otherwise.} \end{cases}$

   (a) **[10 points]** Find $P\{-1 \le X \le 1\}$. Express your answer in terms of K and numerical values obtained from the table.

   (b) **[10 points]** Determine the numerical value of K. **DO NOT go back and substitute this numerical value into your answer to part (a).**

   (c) **[5 points]** Determine the numerical value of $E[X]$.

4. **[25 points]** Let $Y = (1 + X)^5$, where X is uniformly distributed over the interval $[0, 1]$.

   (a) **[10 points]** Find the CDF of Y.

   (b) **[5 points]** Find the pdf of Y.

   (c) **[10 points]** Find the value of $E[Y]$.