University of Illinois Spring 2008

ECE 313: Hour Exam I

1. [30 points] Let A, B, and C denote three events defined on a sample space Ω, and suppose that $P(A) = 0.5, P(B) = 0.4, P(C) = 0.3, P(B^c \cap C) = 0.2$, and $P(A \cap B^c \cap C^c) = 0.1$.

   Find the following probabilities: $P(B \cap C)$, $P(B \cap C^c)$, $P((A \cup B \cup C)^c)$, $P(A^c \cap B)$, and $P(B^c \cap C^c)$.

   If any probability cannot be computed from the given data, check the corresponding box and leave the answer area blank.

2. (a) [10 points] If $\mathcal{X}$ is a Poisson random variable with mean 4, what is $\mathsf{var}(2 + 3\mathcal{X})$?

   (b) [10 points] Let $\mathcal{Y}$ be a negative binomial random variable with parameters (n, p) where n is known, but the value of p is unknown. It is observed that $\{\mathcal{Y} = k\}$. What is the maximum-likelihood estimate $\hat{p}$ of the parameter p?

3. (a) [8 points] Dilbert has 3 coins in his pocket, 2 of which are fair coins while the third is a biased coin with $P(H) = p \neq \frac{1}{2}$. The probability that a coin chosen at random from his pocket will land Tails is $\frac{7}{12}$. What is the value of p?

   (b) [18 points] Let A and B denote events defined on a sample space. Given that $P(A) = 3/5, P(B) = 2/5$, and $P(B|A) = 1/3$, find $P(A|B), P(A^c \cup B^c)$ and $P(B^c|A^c)$.

4. [24 points] At the Democratic National Convention, Hillary Clinton and Barack Obama have equal numbers of delegates committed to them, and neither candidate can win the nomination on a ballot. In desperation, the Convention decides that the two candidates shall debate each other and the winner shall be the nominee of the Democratic Party. On a debate, Clinton wins (event H) with probability $P(H)$, and Obama wins (event B) with probability $P(B)$. A draw (event D) occurs (that is, neither wins) with probability $P(D) = 1 - P(H) - P(B) > 0$. In case of a draw, another debate is held. Successive debates can be regarded as independent trials, and continue until either event H or B occurs, and the Democratic nominee is chosen.

   Express the answers to the following questions in terms of $P(H)$ and $P(B)$.

   (a) [8 points] What is the probability that Hillary is the Democratic nominee?

   (b) [8 points] Given that no more than 5 debates were held, what is the conditional probability that Obama won the nomination?

   (c) [8 points] What is the expected number of debates at the Democratic National Convention?