Assigned: Wednesday, March 12
Due: Wednesday, March 19
Reading: Yates and Goodman: Chapter 2.4 and 4
Reminder: No class on Friday March 14 in honor of Engineering Open House
Problems:

1. Which of the following are valid cumulative probability distribution functions (CDFs) ? For those that are not valid CDFs, state at least one property of CDFs which is not satisfied. For those which are valid CDFs, compute $P\{|\mathbf{X}| > 0.5\}$.

(a) $F_{\mathbf{X}}(u) = \begin{cases} 0, & u < 1, \\ 2u - u^2, & 1 \le u \le 2, \\ 1, & u > 2. \end{cases}$ **(b)** $F_{\mathbf{X}}(u) = \begin{cases} (1/2)\exp(2u), & u < 0, \\ 1 - (1/4)\exp(-3u), & u \ge 0. \end{cases}$

(c) $F_{\mathbf{X}}(u) = \begin{cases} (1/2)\exp(2u), & u \le 0, \\ 1 - (1/4)\exp(-3u), & u > 0. \end{cases}$

2. Let $\mathbf{X}$ denote the number of hours that a student works on ECE 340 each week. It is known that $\mathbf{X}$ is a *mixed* random variable with cumulative probability distribution function (CDF) $F_{\mathbf{X}}(u)$ given by

$$F_{\mathbf{X}}(u) = \begin{cases} 0, & u < 0, \\ (1+u)/8, & 0 \le u < 1, \\ 1/2, & 1 \le u < 2, \\ 1/2 + u/8, & 2 \le u < 4, \\ 1, & u \ge 4. \end{cases}$$

Find the probability that the student

(a) works for exactly 2 hours, **(b)** works for more than 2 hours,
(c) works for less than 2 hours, **(d)** works for exactly 3 hours,
(e) works for more than 1/2 but less than 3 hours,
(f) works for more than 2 hours given that the student works at all, i.e. find $P\{\mathbf{X} > 2 | \mathbf{X} > 0\}$.
(g) Find $E[\mathbf{X}]$.

3.(a) Yates and Goodman, Problem 2.5.11 on page 83. If you decided a long time ago that Y&G was not a book to be put down lightly but one to be thrown with great force, the problem asks you to prove that $E[\mathbf{X}] = \sum_{k=0}^{\infty} P\{\mathbf{X} > k\}$ for a discrete random variable $\mathbf{X}$ that takes on nonnegative integer values only.

(b) Find $P\{\mathbf{X} > k\}$ for k = 0, 1, 2, … for a geometric random variable $\mathbf{X}$ with parameter p. Substitute your answers in the formula of part (a) and give yet another derivation of the result that $E[\mathbf{X}] = 1/p$.

4. Which of the following are valid probability density functions? Assume that the functions are zero outside the ranges specified. For those which are not valid pdfs, state at least one property of pdfs which is not satisfied. Also, state whether there exists a constant C such that Cf(u) is a valid pdf even though f(u) is not.

(a) $f(u) = |u|$ for $|u| < 1$. **(b)** $f(u) = 1 - |u|$ for $|u| < 1$.
(c) $f(u) = \ln u$ for $0 < u < 1$, **(d)** $f(u) = \ln u$ for $0 < u < 2$. Hint: ln u can be integrated by parts
(e) $f(u) = 2u$ for $0 < u < 1$. **(f)** $f(u) = (2/3)(u - 1)$ for $0 < u < 3$.
(g) $f(u) = \exp(-2u)$, $0 < u < \infty$, **(h)** $f(u) = 4\exp(-2u) - \exp(-u)$, $0 < u < \infty$.

5. The random variable **X** has probability density function

$$f_{\mathbf{X}}(u) = \begin{cases} \alpha(1-u), & 0 < u < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $P\{6\mathbf{X}^2 > 5\mathbf{X} - 1\}$.

(b) Find $F_{\mathbf{X}}(u)$. Be sure to specify the value of $F_{\mathbf{X}}(u)$ for all u.

6. The weekly demand (measured in thousands of gallons) for gasoline at a rural gas station is a random variable **X** with probability density function

$$f_{\mathbf{X}}(u) = \begin{cases} 5(1-u)^4, & 0 < u < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Let C (in thousands of gallons) denote the capacity of the tank (which is re-filled weekly.)

(a) If C = 0.5, (i.e., the tank holds 500 gallons) and **X** happens to have value 0.68 one particular week, (e.g. 680 people show up each wanting to purchase a gallon of gas for their snowblowers or lawnmowers), can the gas station satisfy the demand that week? That is, can the gas station supply gasoline to all those who want to buy it that week?

(b) If C = 0.5 and **X** happens to have value 0.43 some other week, can the gas station satisfy the demand during this other week? That is, can the gas station supply gasoline to all those who want to buy it that week?

(c) If C = 0.5, what is the *probability* that the weekly demand for gasoline can be satisfied? Note that if your answer is (say) 0.666…, then, in the long run, the gas station can supply the weekly demand two weeks out of three.

(d) What is the minimum value of C required to ensure that the probability that the demand exceeds the supply is no larger than 10^{-5} ?