University of Illinois | **Problem Set #7** | **ECE 313**

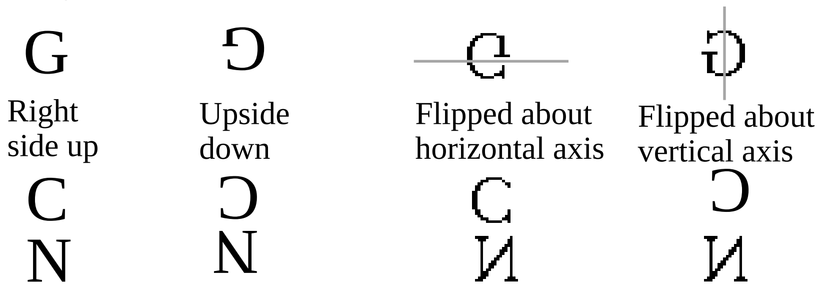
of Illinois | | **Spring 2003**

Assigned: Wednesday, March 5
Due: Wednesday, March 12
Reading: Yates and Goodman: Chapter 4
Problems:

1. ["The Chattanooga Choo-Choo"] Two of the eleven letters in a road sign that reads CHATTANOOGA have fallen down. Assume that each pair of letters is equally likely to have fallen down. A drunk randomly puts the fallen letters back into the two empty slots, possibly interchanging the positions of the letters, and possibly putting the letters back upside down. Thus, all eight possibilities corresponding to the three binary choices

{letters put back in correct position or interchanged}
{left-hand letter upside down or rightside up}
{right-hand letter upside down or rightside up}

are equally likely. Note: The meaning of upside down is illustrated in the diagram below. Be sure that you understand how all the letters in CHATTANOOGA will look upside down



(a) What is the probability that the sign still **reads** CHATTANOOGA?
(b) What is the probability that all the letters **seem** to be correct side up but the sign does not read CHATTANOOGA?
(c) What is the probability that only one letter **seems** to be upside down?
(d) What is the probability that two letters **seem** to be upside down?
(e) Given that all the letters appear to be right side up, what is the (conditional) probability that **at least one** vowel fell down?
(f) A designated driver, who knows that the sign is supposed to read CHATTANOOGA, observes the restored sign. What is the probability that the driver can **correctly** identify (without any possibility of making a mistake) which letters had fallen down?

2. ["Baby needs new shoes!"] The dice game of craps begins with the player (called the shooter) rolling two fair dice. If the result is a 2, or 3, or 12, the shooter loses, while if the result is a 7 or 11, the shooter wins. The shooter who rolls any of 4, 5, 6, 8, 9, 10 has neither won nor lost (as yet). What happens then is discussed in parts (b) and (c).
(a) What is the probability that the shooter loses on the first roll? What is the probability that the shooter wins on the first roll?
(b) If the sum of the dice on the **first roll** is any of 4, 5, 6, 8, 9, 10, that number is called the **shooter's point.** For **each** number i in the set {4, 5, 6, 8, 9, 10}, find the probability that the shooter's point is i. I need six answers here, folks!
(c) Suppose that the shooter's point is i where i is some number in {4, 5, 6, 8, 9, 10}. The shooter now rolls the two dice again. If the result is a 7, the shooter loses (craps out.) If the result is i, the shooter wins (this is referred to as making the point). If the result is neither i nor 7, the shooter rolls again. This process continues until the shooter either makes the point or craps out. Given that the shooter's point is i, what is the conditional probability that the shooter makes the point? Naturally, the answer depends on i, so here too, I need six answers.
(d) Use the above results to compute the probability of winning at craps.
(e) Given that the shooter's point is 8, what is the probability that the shooter makes it "the hard way," that is, by rolling two fours? Generally, bets are offered at 10-to-1 odds that the shooter makes the point 8 the hard way. That is, if you bet $1, you win $10 (plus your $1

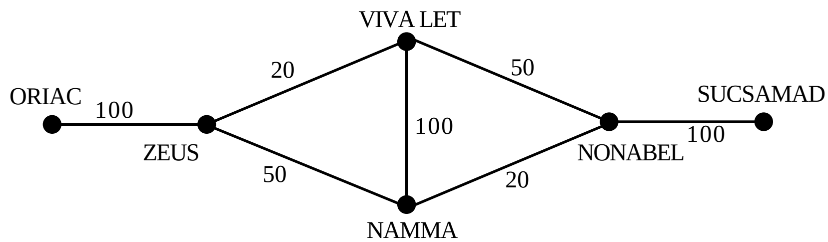
back!) if the shooter makes 8 the hard way; and you lose the $1 that you bet if the shooter craps out or makes 8 by rolling 2-6, 3-5, 5-3, or 6-2). In the long run over many such bets, do you expect to make money, or lose money, or come out even?

3. ["Reach out and touch someone, Call up and just say Hi!"] MiddleEast Bell, a division of Psingular Corp., has built a telephone network as shown below. Terrorists attack each of the seven links. The attacks may be considered to be independent events, and the attack on a link succeeds in severing the link with probability p. If a link is severed, switches automatically re-route calls so as to avoid the failed link (if possible).

(a) What is the probability of being able to call from ORIAC to SUCSAMAD?

(b) Given that it is possible to call from ORIAC to SUCSAMAD, what is the conditional probability that the ZEUS to NAMMA link is in working condition?

(c) The link capacities (i.e., the numbers of telephone calls that the links can carry (in either direction)) are as marked on the diagram. Let **X** denote the number of calls that can be made from ORIAC to SUCSAMAD. Find the pmf and the expected value of **X**.



4. ["Somebody's gonna Lotto, Might as well be you…"] Participants in a lottery buy tickets for $1 each. Each ticket bears one of the numbers 1 to 32. One of these 32 numbers is drawn at random, and all holders of tickets with the winning number are paid $29 per ticket (i.e. the ticket price plus winnings of $28).

(a) If you buy a ticket numbered 7, what is the probability that you win? If you buy a ticket for each drawing of the lottery, what is the **average** number of drawings until you win? Is this average number calculation valid only if you buy a ticket numbered 7 each time, or can you vary your choice with each drawing?

(b) What is your average win (or loss) per drawing?

(c) If you buy 32 tickets numbered 1 through 32, you will always hold a winning ticket. But, how much money do you lose on each drawing? What is your loss **per ticket?**

(d) In response to complaints that the lottery payoffs are too low, the lottery administrator claims that *on average,* number 7 (say) wins at least once in 22 drawings while he is paying out $28 on winning tickets. Thus, the lottery is actually biased in the player's favor! He backs up his claim by offering the following even-money bet: if 7 does not **win** in the next 22 drawings, he will pay you $100, while if 7 does win, then you will pay him $100. You accept this wager and sure enough, 7 wins on the twelfth drawing. "Pay up" says the lottery administrator, and after collecting, asks if you want to bet that 7 does not win on the *next* 22 drawings. You accept, and this time, 7 does not win in the next 22 drawings and you get your money back. You play this game with the lottery administrator many times, and find that he is right! On roughly half of these new-fangled bets, 7 does not occur in the next 22 drawings and you collect, but on the other half of your bets, 7 does occur sooner, and you have to pay up. But, is the bet *perfectly* fair (in the sense that the expected win/loss per bet is 0)? Does it prove the lottery administrator's claim that 7 wins at least once (on average) in 22 drawings? If you agree with the lottery administrator, how do you reconcile this with part (a) where you found (I hope!) that 7 wins once in 32 drawings (on average?)