Assigned: Wednesday, February 12
Due: Wednesday, February 19
Reading: Yates and Goodman: Chapter 2 except Sections 2.4 and 2.9
Noncredit Exercises: (Do not turn these in) p. 55: Quiz 2.3; p. 67: Quiz 2.6; pp. 80-86: Problems 2.3.1 – 2.3.12, 2.8.1 – 2.8.12

Problems:

1. Eight people hold reservations for travel in a 5-passenger limousine from Champaign to St. Louis. The number of persons who actually show up to travel can be modeled as a binomial random variable **X** with parameters $(8, \frac{1}{2})$. If more than 5 people show up, only the first 5 get to go, and the rest are left behind. What is the average number of passengers who are left behind?

2. Use a spreadsheet/Mathematica/MATLAB for this problem.
Let A denote an event of probability p.

(a) For p = 0.1, 0.25. 0.4, 0.5, 0.6, 0.75, and 0.9, find the numerical values of the probabilities that A occurs 0, 1, 2, … , 10 times on 10 independent trials of the experiment.

(b) You have, in effect, computed the probability mass function for a binomial random variable **X** with parameters (10, p) for seven choices of p. For each value of p, draw a bar graph of the pmf.

(c) What is the relationship between the pmfs for the cases p = 0.1 and p = 0.9? for the cases p = 0.25 and p = 0.75? for the cases p = 0.4 and 0.6?

(d) From each of the seven graphs of part (b), find the value of k for which P{**X** = k} is maximum. Compare your results to our prediction of Monday's lecture.

(e) Prove that for a binomial random variable with parameters (n, p), the mean and the mode differ by at most 1.

3. ["No, dear; Cheetos is not a vegetable…"] Let A, B, and C denote the events that your mother serves respectively asparagus, broccoli, and cauliflower for dinner. From (bitter?) experience you know that she serves only one vegetable each day and that P(A) = 0.2, P(B) = 0.5, and P(C) = 0.3. Each day is an independent trial: that is, your mother, a lady of formidable temperament albeit limited culinary skills, makes *independent* decisions (i.e., without taking into account your opinion that Cheetos is *the* perfect accompaniment to any entreé) about the vegetable to serve each day. Over a three day period, what is the probability that

(a) she serves the same vegetable on all three days ?

(b) she serves the same vegetable exactly two days out of three ?

(c) she serves different vegetables on the three days ?

4. In the game of Chuck-A-Luck often played in MidWestern carnivals and fairs, a player bets money (the stake) on one of the numbers 1, 2, 3, 4, 5, 6. Three fair dice are rolled (so that the sample space has $6^3 = 216$ equally likely outcomes in it). If the number chosen shows up on one, two, or all three of the dice, the player ***wins*** respectively one, two, or three times the money that was staked; and, of course, the original money staked is also returned to the player (the return of the stake money is not counted as part of the winnings). If the chosen number shows up on none of the dice, the player ***loses*** the money that was staked. **X** is the random variable that denotes the amount of money ***won*** for a $6 bet.

(a) What are the possible values that **X** can take on? Remember that negative values of **X** denote losses.

(b) What is the probability mass function (pmf) of **X**?

(c) What is the expected value of **X**?

(d) ["Always go out a winner"] A player splits his $6 bet into a $1 bet on each of the six numbers with the idea that at least one, and possibly as many as three, of his bets will be winners. Let **Y** denote his winnings. What are the values taken on by **Y**? What is the expected value of **Y**? Compare your answer to the expected value of **X** found in part (c).

5. Let **X** denote a Poisson random variable with parameter λ.

(a) Show that P{**X** is even} = $\exp(-\lambda)\cosh(\lambda)$. Don't forget that 0 is an even integer!

(b) In Problem 6 of Problem Set #3, you proved (I hope!) that the probability that a binomial random variable with parameters (n,p) has even value is $[1+(1-2p)^n]/2$. For large n and small p, show that $[1+(1-2p)^n]/2 \approx \exp(-np)\cosh(np)$, which is consistent with part (a).

6. Suppose that 105 passengers hold reservations for a 100–seat flight from Chicago to Champaign. The number of passengers showing up for the flight can be modeled as a binomial random variable **X** with parameters (105,0.9).

(a) Find the probability that all passengers who show up get seats, i.e. find P{**X** $\leq$ 100}.

(b) Explain why the number of no-shows can be modeled as a Poisson random variable **Y**, and compute the value of the parameter λ.

(c) Compute the probability that all passengers who show up get seats based on this Poisson model, i.e find P{**Y** $\geq$ 5}, and compare to the "more exact" answer of part (a).