Assigned: Wednesday, February 5
Due: Wednesday, February 12
Reading: Yates and Goodman: Sections 2.1-2.3, 2.5-2.8
Noncredit Exercises: (Do not turn these in) p. 48: Quiz 2.2; p. 64: Quiz 2.5; p. 67: Quiz 2.7; p. 74, Quiz 2.8; pp. 80-86: Problems 2.2.1, 2.2.5, 2.3.5, 2.5.2-2.5.6, 2.6.2, 2.7.4, 2.7.6, 2.8.1-2.8.6

Problems:
1. ["Eat your broccoli, dear; it's good for you"] Your mother has bought three servings of broccoli and two servings of cauliflower for next week (Monday through Friday) and will serve one vegetable on each of these days.

(a) Define an appropriate sample space   and state how many outcomes are in   . Assume that all outcomes are equally likely.
(b) What is the probability of having broccoli on Monday ?
(c) What is the probability of having broccoli on Monday and Friday ?
(d) What is the probability of having respectively broccoli, broccoli, cauliflower, broccoli, and cauliflower on Monday, Tuesday, Wednesday, Thursday, and Friday ?
    Optional for zero extra credit: Write a 500-word essay explaining why you like dorm food so much.

2. Find $P(A \cup (B^c \cup C^c)^c)$ in each of the following four cases:
(a) A, B, and C are mutually exclusive events and $P(A) = 1/3$.
(b) $P(A) = 2P(BC) = 4P(ABC) = 1/2$.
(c) $P(A) = 1/2$, $P(BC) = 1/3$, and $P(AC) = 0$.

(d) $P(A^c \cup (B^c \cup C^c)) = 0.6$.

3. The experiment consists of picking a student from the set of all UIUC students registered this semester. It is **not** necessary to assume that all students are equally likely to be picked, but you may make this assumption if it makes you feel happier and more confident.
(a) Let A and B denote the events that the student picked has had respectively four years of science (FYS) and calculus in high school. Let $P(A) = 0.45$ and $P(B) = 0.35$. If the probability that the student had neither FYS nor calculus is 0.3, what is the probability that the student had both FYS **and** calculus? What is the probability that the student had FYS but **not** calculus ?
(b) Let C denote the event that the student is registered in ECE 313, and let A and B be as in part (a). Suppose that $P(A \cup B \cup C)=0.002$. What is the probability that the student picked is not registered in ECE 313, but did have both FYS **and** calculus ? If the probability that the student picked is registered in ECE 313, and has had either FYS or calculus (but not both) is 0.004, and if students who had neither FYS nor calculus did not register in ECE 313, what is $P(C)$?
(c) Using the data given in parts (a) and (b), which of the following probabilities can you compute? It is not necessary to actually compute each probability.

 $P(A \cup C)$, $P(A \cup B \cup C)$, $P(A \cup B \cup C^c)$, $P(A^cB^cC^c)$, $P(A^cBC^c)$, $P(ABC^c)$

4.(a) Fred, Wilma, Barney, and Betty take turns (in that order) tossing a coin that has
P(Heads) = p, $0 < p < 1$. The first one to toss a Head wins the game. Calculate the win
probabilities P(F), P(W), P(Ba) and P(Be) of the players and show that
   (i) P(F) > P(W) > P(Ba) > P(Be).
and
   (ii) P(F) + P(W) + P(Ba) + P(Be) = 1.

(b) Ω = {0, 1, 2, … } is a countably infinite sample space with $P\{n\} = \dfrac{(\ln 2)^n}{2(n!)}$ for all n $\geq$ 0.
(Remember that 0! = 1).
   (i) Verify that $P(\Omega) = 1$ for this probability assignment.
   (ii) Show that P(outcome is an even number) = 5/8. Remember: 0 is an even number!

5. The prospectus of GoGoDotCom Inc., an investment management service, states that their
goal is to double the value of their clients' investments in a week via day trading of Internet
stocks. (The Securities and Exchange Commission insists, as usual, that a disclaimer be
included that there is no guarantee that the goal will be met.) The TV commercials proclaim
"On average, our clients triple their money in five weeks!" You decide to invest $32 (hey,
you are a student on a tight budget) with GoGoDotCom Inc. for a period of five weeks.
Let **X** denote the value (in dollars) of your investment at the end of this period.
Now suppose that GoGoDotCom Inc.has a 50% chance of doubling your investment and
50% chance of losing half your investment. That is, if you invest $C with them, then, a
week later, your investment is equally likely to be worth $2C or $C/2. Assume that each
week's performance is an independent trial that doubles or halves the value that your
investment had at the beginning of the week.
(a) What are the possible values of **X**?
(b) What is the pmf of the random variable **X**?
(c) What is the expected value of **X**? Is the TV commercial an accurate statement?
(d) What is the probability that you will lose money on this investment? i.e. find P{**X** < 32}.
(e) An investment of $32,000 with GoGoDotCom Inc. would be worth $1000**X** in five
weeks. Assuming that you have the money, would you be willing to make such an
investment? Why or why not? Would you be willing to borrow the money from your
parents to make the investment? How about borrowing the money from a loan shark?

6. Let **X** denote a binomial random variable with parameters (N, p). What is the probability
that **X** is an even integer? Remember that 0 is an even integer.
[Hint: What is $(x+y)^N + (x - y)^N$?]