Assigned: Wednesday, January 22
Due: Wednesday, January 29
Reading: Yates and Goodman: page x, and Sections 1.1-1.4
Noncredit Exercises: (Do not turn these in) pp. 6-7: Quiz 1.1, pp. 37-38: Problems 1.2.1-1.2.6.

Let E, F, and G denote events. Express the following events in terms of E, F, G, and the , $\cap$, and ^{c} operators: *only* E occurs (the other two do not); *at least one* event occurs; *at least two* of the events occur; *all three* events occur; *none* of the events occurs; *at most one* of the events occurs; *exactly* two of events occur.

Problems: Most of the topics covered in this problem set will be needed *after* the drop date. Hence, this problem set (which is based entirely on material covered in the *prerequisites* to this course) is intended as a review and as a diagnostic aid for identifying areas that you may need to review one more time before starting the course.

1.(a) Prove that $1 + x + x^2 + \ldots + x^{n-1} = \dfrac{1 - x^n}{1 - x}$ for $x \neq 1$ and integer $n \geq 1$.

(b) Compute the first four terms of the Taylor series (a.k.a. 3rd degree Taylor polynomial) for $f(x) = (1 + x)^n$.

(c) Repeat part (b) for $g(x) = (1 - x)^n$.

(d) True or false? The Taylor *series* for $(1 + x)^n + (1 - x)^n$ contains only the even powers of x

2.(a) Find the limit of $\dfrac{1}{[\sin x]^2} - \dfrac{1}{x^2}$ as x approaches 0. Use your calculator to evaluate this function for *small* values of x say, $x = 10^{-1}$, $x = 10^{-2}$, $x = 10^{-3}$, etc. Does the function seem to be approaching a limit, and if so, what do you think is the limit? Now, use what you have learned about limits in calculus to find $\lim_{x \to 0} \dfrac{1}{[\sin x]^2} - \dfrac{1}{x^2}$ analytically.

(Hint: the answer is not 0, or 1, or ∞).

(b) Find all the maxima (maxima = plural of maximum) of $f(x) = x^{25}(1.00001)^{-x}$ for $x > 0$.

3.(a) What is the value of $\int_{-2}^{1} |x|\,dx$? the value of $\int_{-2}^{1} x(1-x)^{19}\,dx$?

(b) Prove or disprove: there exists a function f(x) satisfying **both** of the following two conditions:

(i) $f(x) \geq 0$ for all real numbers x in the range $-2 \leq x \leq 1$,

(ii) $\int_{-2}^{1} f(x)dx < 0$. (Hint: Does either function of part (a) satisfy both conditions?)

(c) Let $\frac{d}{dx}f(x) = g(x)$ for $-\infty < x < \infty$. Which of the following statements are true for all x, $-\infty < x < \infty$? In parts (iv)-(vi), C denotes an arbitrary constant.

(i) $\frac{d}{dx}f(-x) = g(-x)$. (ii) $\frac{d}{dx}f(x^2) = 2x\,g(x^2)$. (iii) $\frac{d}{dx}\exp(f(x^2)) = \exp(f(x^2))\,g(x^2)$.

(iv) $\int g(-x)dx = -f(-x) + C$. (v) $\int g(x^2)dx = f(x^2)/(2x) + C$. (vi) $\int \frac{g(x)}{f(x)}dx = \ln(f(x)) + C$.

(d) Evaluate $\int_{0}^{\infty} x \cdot \exp(-x^2/2)dx$.

4.(a) Integrate $f(x, y) = \max(x,y)$, over the region $\{(x,y) : 0 < x < 2, 0 < y < 1\}$.

(b) Compute the integral of $(x^2 + y^2)^{-2}$ over the region $\{(x,y) : x^2+y^2 > 2\}$.

5. Convolve the functions $f(t) = u(t) - u(t-T)$ and $g(t) = u(t) \cdot \exp(-t)$ where u(t) is the *unit step function*. You must specify the value of $h(t) = f*g$ for all t, $-\infty < t < \infty$.