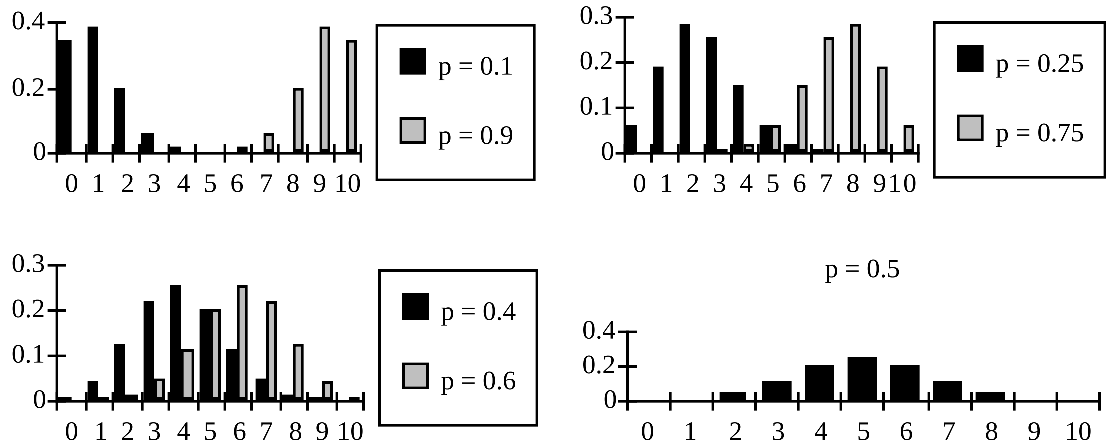
1. **Y** = 1, 2, 3 passengers are left behind according as **X** = 6, 7, or 8. Since **X** takes on values 6, 7, 8 with with probabilities $\frac{28}{256}$, $\frac{8}{256}$, $\frac{1}{256}$ respectively, we readily find that $E[\mathbf{Y}] = \frac{1\times 28 + 2\times 8 + 3\times 1}{256} = \frac{47}{256}$.

2.(a), (b)



(c) From the table, the probabilities for any given value of p and 1–p are just "reverses" of each other in the sense that P{**X** = k} for probability p is just P{**X** = 10–k} for probability 1–p.

(d) For p = 0.1, 0.25. 0.4, 0.5, 0.6, 0.75, and 0.9, P{**X** = k} is largest for k = 1, 2, 4, 5, 6, 8, and 9 respectively exactly as predicted by the theory (would I lie to you?)

(e) The mean is np and the mode is $\lfloor (n+1)p \rfloor$, so the difference between the two values is ≤ p ≤ 1. Why?

3.(a) P(same on all three days) = $(0.2)^3 + (0.5)^3 + (0.3)^3 = 0.16$.

(b) P(same on two of three days) = $3 \times \{(0.2)^2 \times [1 - 0.2] + (0.3)^2 \times [1 - 0.3] + (0.5)^2 \times [1 - 0.5]\} = 0.66$.

(c) P(different on all three days) = $3![0.2 \times 0.5 \times 0.3] = 0.18$. Alternatively, we can compute this as $1 - 0.16 - 0.66$. (Why?)

4.(a) **X** takes on values –6, 6, 12, 18.

(b) If $6 is bet on i, then you lose it if all three dice show one of the 5 non-i numbers. Hence P{**X** = –6} = $5^3/6^3 = \frac{125}{216}$. On the other hand, you win $6 if one of the three dice shows i and the other two have non-i numbers. Hence, P{**X** = 6} = $3\bullet(1\bullet 5^2)/6^3 = \frac{75}{216}$. By a similar argument, P{**X** = 12} is $3\bullet(1^2\bullet 5)/6^3 = \frac{15}{216}$, and P{**X** = 18) = $1^3/6^3 = \frac{1}{216}$. Sanity check: 125 + 75 + 15 + 1 = 216, so we have not left anything out.

(c) $E[\mathbf{X}] = \sum u\bullet p(u) = \frac{125\bullet(-6) + 75\bullet 6 + 15\bullet 12 + 1\bullet 18}{216} = -\frac{102}{216} = -\frac{17}{36}$ i.e. a loss of roughly 47¢ per game.

(d) The chance of the three dice showing three different numbers is $\frac{6\bullet 5\bullet 4}{216} = \frac{5}{9}$. In this case, you come out even since you win $3 on the three numbers showing, but lose $3 on the three no-shows. The chance that the three dice show the same number is $6\bullet\frac{1}{216} = \frac{1}{36}$ in which case you win $3 on the winning number but lose $5 on the 5 no-shows for a net loss of $2. The probability that exactly two numbers are identical is thus $\frac{15}{36} = \frac{5}{12}$ in which case, you win $2 on the pair and $1 on the singleton, but lose $4 on the other no-shows for a net loss of –1. Thus, **Y** is a random variable taking on values 0, –1, –2, with probabilities as found above. (Remind me once again why you are bothering to play this game at all?), and its expected value is

$E[\mathbf{Y}] = \frac{0\cdot 20 - 1\cdot 15 - 2\cdot 1}{36} = -\frac{17}{36}$ just as before. Splitting your bet six ways has no effect on your losses!

Exercise: Would it be better to bet on 2 or 3 or 4 or 5 numbers (in equal shares) instead?

5.(a) $\sum_{k \text{ even}} P\{\mathbf{X} = k\} = \exp(-\lambda)\sum_{k=0}^{\infty} \lambda^{2k}/2k! = \exp(-\lambda)\cosh(\lambda)$ via the known series for cosh. Geez, that was easy!

(b) $[1+(1-2p)^n]/2 = [1+(1-2np/n)^n]/2 \approx [1+\exp(-2np)]/2 = \exp(-np)[\exp(np)+\exp(-np)]/2$
$= \exp(-np)\cosh(np) = \exp(-\lambda)\cosh(\lambda)$ on setting $\lambda = np$. He's losing his touch; that was even easier!

6.(a) $P\{\mathbf{X} \le 100\} = 1 - P\{\mathbf{X} > 100\}$
$= 1 - [P\{\mathbf{X} = 105\} + P\{\mathbf{X} = 104\} + P\{\mathbf{X} = 103\} + P\{\mathbf{X} = 102\} + P\{\mathbf{X} = 101\}]$

$$= 1 - \left[(0.9)^{105} + \binom{105}{1}(0.9)^{104}(0.1) + \binom{105}{2}(0.9)^{103}(0.1)^2 + \binom{105}{3}(0.9)^{102}(0.1)^3 + \binom{105}{4}(0.9)^{101}(0.1)^4\right]$$

$= 0.983283...$

(b) If $\mathbf{X}$ is a binomial random variable with parameters (n, p), then $\mathbf{Y} = n - \mathbf{X}$ is a binomial random variable with parameters (n, 1–p). Thus, the number of no-shows is a binomial random variable with parameters (105,0.1). Since n is large and p is small, it is reasonable to approximate this as a Poisson random variable with parameter $\lambda = np = 10.5$.

(c) $\{\mathbf{Y} \ge 5\} = 1 - P\{\mathbf{Y} < 5\} = 1 - [P\{\mathbf{Y} = 0\} + P\{\mathbf{Y} = 1\} + P\{\mathbf{Y} = 2\} + P\{\mathbf{Y} = 3\} + P\{\mathbf{Y} = 4\}]$
$= 1 - \exp(-10.5)[1 + (10.5) + (10.5)^2/2 + (10.5)^3/6 + (10.5)^4/24] = 0.978906\ldots$ which is not bad…