1.(a) The outcomes are all the vectors of length 5 in which 3 entries are broccoli and 2 are cauliflower. The days to serve broccoli can be chosen in $\binom{5}{3} = 10$ ways, so $|\Omega| = 10$.

(b) Broccoli will be served on two other days besides Monday, and these days can be chosen in $\binom{4}{2} = 6$ ways. Hence, P(broccoli on Monday) = 6/10 = 3/5.

(c) The third broccolous day can be any of 3, and hence probability is 3/10. **(d)** 1/10.

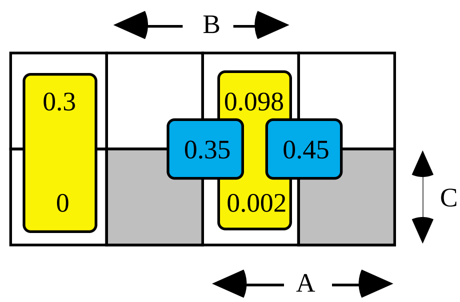
2. $P(A \quad (B^c \quad C^c)^c) = P(A \quad (B \cap C))$ by DeMorgan's theorem.

(a) $P(B \cap C) = P(\quad) = 0$, and therefore $P(A \quad (B \cap C)) = P(A \quad\quad) = P(A) = 1/3$.

(b) $P(A \quad (B \cap C)) = P(A) + P(B \cap C) - P(A \cap B \cap C) = 1/2 + 1/4 - 1/8 = 5/8$.

(c) $P(A \quad (B \cap C)) = P(A) + P(B \cap C) - P(A \cap B \cap C) = 1/2 + 1/3 - 0 = 5/6$. (Why?)

(d) $(A^c \cap (B^c \quad C^c))^c = A \quad (B^c \quad C^c)^c$ by DeMorgan's theorem. Hence, $P(A \quad (B^c \quad C^c)^c) = 1 - P(A^c \cap (B^c \quad C^c)) = 1 - 0.6 = 0.4$.



3.(a) From DeMorgan's theorem and $P(A^cB^c) = 0.3$, we get that $P(A \quad B) = 0.7$. $P(AB) = P(A) + P(B) - P(A \quad B) = 0.1$. $P(AB^c) = P(A) - P(AB) = 0.35$.

(b) $P(ABC^c) = P(AB) - P(ABC) = 0.098$. Note that C can be partitioned into ABC, $(A \quad B)C$ and A^cB^cC, where $(A \quad B)C$ is the shaded set in the right-hand figure shown above. Since $P(A^cB^cC) = 0$, we get that $P(C) = P(ABC) + P(A \quad B)C) = 0.006$.

(c) $P(A^cBC^c)$ and $P(A \quad C)$ cannot be computed.

4.(a) Let q = 1–p. Then,

$$P(F) = p + q^4p + q^8p + \ldots = p[1 + q^4 + q^8 + \ldots] = \frac{p}{1 - q^4}$$

$$P(W) = qp + q^5p + q^9p + \ldots = qp[1 + q^4 + q^8 + \ldots] = \frac{pq}{1 - q^4} = q \bullet P(F)$$

$$P(Ba) = q^2p + q^6p + q^{10}p + \ldots = q^2p[1 + q^4 + q^8 + \ldots] = \frac{q^2p}{1 - q^4} = q^2 \bullet P(F)$$

$$P(Be) = q^3p + q^7p + q^{11}p + \ldots = q^3p[1 + q^4 + q^8 + \ldots] = \frac{q^3p}{1 - q^4} = q^3 \bullet P(F)$$

Since q < 1, P(F) > P(W) > P(Ba) > P(Be). Also,

$$P(F) + P(W) + P(Ba) + P(Be) = P(F)\bullet[1 + q + q^2 + q^3] = P(F)\bullet\frac{q^4 - 1}{q - 1} = \frac{p}{1 - q^4} \bullet \frac{q^4 - 1}{q - 1} = 1.$$

(b) Let x = ln 2. Then, $P(\Omega) = \sum_{n=0}^{\infty} \frac{(\ln 2)^n}{2(n!)} = 2^{-1} \sum_{n=0}^{\infty} \frac{x^n}{n!} = 2^{-1} \bullet \exp(x) = 1.$

$$P(\text{outcome is an even number}) = \sum_{n=0}^{\infty} \frac{(\ln 2)^{2n}}{2(2n!)} = 2^{-1} \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n!)} = 2^{-1} \bullet \cosh(x) = \frac{5}{8} \text{ since } x = \ln 2.$$

5.(a) The number of weeks that your investment doubles in value is a *binomial* random variable **Y** with parameters (5,1/2). Since the investment halves in value during the remaining 5–**Y** weeks, and each halving cancels one doubling, we have that $\mathbf{X} = 32 \cdot 2^{2\mathbf{Y}-5}$. The possible values of **X** are 1, 4, 16, 64, 256, and 1024, corresponding to **Y** = 0, 1, 2, 3, 4, 5 respectively.

(b) P{**X** = 1} = P{**Y** = 0} = 1/32. P{**X** = 4} = P{**Y** = 1} = 5/32. P{**X** = 16} = P{**Y** = 2} = 10/32. P{**X** = 64} = P{**Y** = 3} = 10/32. P{**X** = 256} = P{**Y** = 4} = 5/32. P{**X** = 1024} = P{**Y** = 5} = 1/32.

(c) E[**X**] = 1•(1/32) + 4•(5/32) + 16•(10/32) + 64•(10/32) + 256•(5/32) + 1024•(1/32) = 97.65625. The TV commercial *understates* the performance — undoubtedly a first!

(d) P{**X** < 32} = P{**X** = 1} + P{**X** = 4} + P{**X** = 16} = 1/2.

(e) There is a 50% chance of losing money on this investment. Most people do not mind making investments if the amount at risk is small but the payoff from a win is enormous, e.g. lottery tickets are a losing bet for most buyers, but people don't mind an almost sure loss of a dollar because all they see is the huge payoff. Matters are considerably different when the amounts at risk are large, and most people tend to choose more conservatively. This explains why the local 7-11 sells lottery tickets but Neiman-Marcus does not.

6. P{**X** is even} = P{**X** = 0} + P{**X** = 2} + … + P{**X** = M} where M = N if N is even and M = N – 1 if N is odd. Hence, P{**X** is even} = $q^N + \binom{N}{2}q^{N-2}p^2 + \ldots + \binom{N}{M}q^{N-M}p^M$. Note that the last term is p^N if N is even, and it is Nqp^{N-1} if N is odd (whatever happened to $\binom{N}{M}$??)

Now, $(q+p)^N = q^N + \binom{N}{1}q^{N-1}p + \binom{N}{2}q^{N-2}p^2 + \ldots + \binom{N}{N-1}qp^{N-1} + p^N$ while

$(q-p)^N = q^N - \binom{N}{1}q^{N-1}p + \binom{N}{2}q^{N-2}p^2 - \ldots + (-1)^{N-1}\binom{N}{N-1}qp^{N-1} + (-1)^N p^N$.

We add these equations together and note alternate terms cancel.. Also if N is odd, the last terms cancel, while if N is even, the next–to–last terms cancel. In either case, we see that the sum is just 2P{**X** is even}, i.e. P{ **X** is even} = $[(q+p)^N + (q-p)^N]/2 = [1 + (1-2p)^N]/2$. Exercise: when is P{ **X** is even} < 1/2?