University of Illinois	**Second Hour Exam Fifty minutes**	**ECE 313 Spring 2003**

1. Let A, B, and C denote events such that

$$P(A) = 1/2,\ P(B) = 1/4, \text{ and } P(C) = 2/3.$$

Suppose that A and B are *mutually exclusive* events, and that A and C are *mutually independent* events. Given that $P(B|C) = 1/4$, find

(a) the probability that at least one of the events A, B, and C occurs.

(b) the probability that at least two of the events A, B, and C occur.

(c) the probability that C did not occur given that exactly one of A and B^c did.

2. **X** denotes a continuous random variable with probability density function (pdf) $f_{\mathbf{X}}(u)$ given by

$$f_{\mathbf{X}}(u) = \begin{cases} (\pi/8)\sin(\pi u/4), & 0 < u < 4, \\ 0, & \text{elsewhere.} \end{cases}$$

Find $P\{3\mathbf{X}^2 < 17\mathbf{X} - 10\}$. You may leave the answer in terms of radicals.

Useful facts: $\cos(\pi/6) = \sin(\pi/3) = \sqrt{3}/2$; $\cos(\pi/3) = \sin(\pi/6) = 1/2$;
$\cos(\pi/4) = \sin(\pi/4) = 1/\sqrt{2} = -\cos(5\pi/4) = -\sin(5\pi/4)$

3. **X** denotes a Gaussian random variable with mean 2 and variance 100.

(a) Use the attached table of $\Phi(x)$ to find $P\{|\mathbf{X}| < 8\}$.

(b) Find the value of $E[(\mathbf{X}-4)^2]$.

(c) Find $f_{\mathbf{Y}}(v)$, the probability density function (pdf) of $\mathbf{Y} = (\mathbf{X} - 2)^2$.
To receive full credit, you must specify the value of $f_{\mathbf{Y}}(v)$ for all v, $-\infty < v < \infty$.

4. Consider a Poisson process with arrival rate 2 per second. Let A denote the event that there is exactly *one arrival* in the time interval (0, T] and B the event that there are *no arrivals* in the time interval (0.5T, 1.5T].

(a) What are the values of P(A) and P(B)?

(b) Find $P(B | A)$.