University of Illinois | **First Hour Exam** | **ECE 313 Spring 2003**

Two versions of the exam, differing in minor ways, were used.

1. **(35 points)** Let A, B, and C denote three events defined on a sample space Ω, and suppose that $P(A) = 0.3$, $P(B) = 0.3$, $P(C) = 0.5$, and $P(A \cap B^c) = P(A^c \cap B^c \cap C) = 0.2$. Find $P(A \cap B)$, $P(A^c \cap B)$, $P((A \quad B \quad C)^c)$, $P(B \cap C^c)$ and $P(C^c \mid (A^c \cap B^c))$.

The other version interchanged events A and C and asked for $P(B \cap C)$ instead of $P(A^c \cap B)$.

2. **(35 points)** Consider 10 repeated independent trials of the experiment of tossing a fair coin.

(a) **(4 points)** If $\mathbf{X}$ denotes the number of tails on the 10 tosses, what is the value of $E[\mathbf{X}]$? If you find the numerical value of $E[\mathbf{X}]$ by using a formula on your sheet of notes, ***explain*** why you believe that it is the correct formula to use in solving this problem.

(b) **(9 points)** Let A be the event that there were exactly 4 Heads on the *first* 6 tosses. What is P(A)?

(c) **(6 points)** What is $E[\mathbf{X}|A]$, the *conditional* expected value of $\mathbf{X}$ given A?

(d) **(16 points)** Let B be the event that there were exactly 2 Heads on the last 6 tosses. What is $P(A \cap B)$? What is $P(A \mid B)$?

The other version interchanged events A and B.

3. **(20 points)** Two of the five letters in a road sign that reads MIAMI have fallen down. Assume that each pair of letters is equally likely to have fallen down. A drunk randomly puts the fallen letters back into the two empty slots, possibly interchanging the positions of the letters, and possibly putting the letters back upside down. Thus, all eight possibilities corresponding to the three binary choices

{letters put back in correct position or interchanged}
{left-hand letter upside down or rightside up}
{right-hand letter upside down or rightside up}

are equally likely.

(a) **(10 points)** What is the probability that the sign still seems to read MIAMI?

(b) **(10 points)** Given that the sign still seems to read MIAMI, what is the probability that the 2 M's fell down?

The other version asked for the conditional probability that the 2 I's fell down.

4. **(10 points)** Let $\mathbf{X}$ denote a Poisson random variable. If $E[\mathbf{X}] = 3$, what is the average value of $(\mathbf{X}-2)^2$?

The other version used $E[\mathbf{X}] = 4$.