

Assigned: Wednesday, April 10, 2002

Due: Wednesday, April 17, 2002

Reading: Ross, Chapter 6 (except Sections 6.6 and 6.8)

Noncredit Exercises: Ross, Chapter 6: Problems 1, 10-15, 20-23

Problems:

1. Ross, Chapter 6, Problem 8
2. Ross, Chapter 6, Problem 9
3. We return to the “random chord” of Problem 1 of Problem Set #11. Yet another way of defining a “random chord” is to choose the midpoint of the chord to be anywhere inside the circle with equal probability. The chord is, of course, perpendicular to the diameter that passes through the chosen point. Thus, let the random point (X, Y) be uniformly distributed on the interior of the circle of unit radius centered at the origin (this region is called the unit disc — nomenclature that might be familiar to DSPists).
 - (a) Find the probability that the length L of the random chord is greater than the side of the equilateral triangle inscribed in the circle.
 - (b) Express L as a function of the random variable (X, Y) and find the probability density function for L .
 - (c) Find the average length of the chord, i.e. find $E[L]$.
4. The jointly continuous random variables X and Y have joint pdf given by

$$f_{X,Y}(u,v) = \begin{cases} 2 \exp -(u + v), & 0 < u < v < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$
 - (a) Sketch the u - v plane and indicate on it the region over which $f_{X,Y}(u,v)$ is nonzero.
 - (b) Find the marginal pdfs of X and Y .
 - (c) Are the random variables X and Y independent ?
 - (d) Find $P\{Y > 3X\}$.
 - (e) For $\alpha > 0$, find $P\{X + Y \leq \alpha\}$.
 - (f) Use the result in part (e) to determine the pdf of the random variable $Z = X + Y$.
5. The jointly continuous random variables X and Y have joint pdf

$$f_{X,Y}(u,v) = \begin{cases} 1/2, & 0 \leq u < 1, 0 \leq v < 1, \text{ and } 0 \leq u + v < 1 \\ 3/2, & 0 \leq u < 1, 0 \leq v < 1, \text{ and } 1 \leq u + v < 2 \\ 0, & \text{otherwise.} \end{cases}$$
 Find $f_X(u)$, $P\{X + Y \leq 3/2\}$ and $P\{X^2 + Y^2 \geq 1\}$.
6. Let X and Y denote *independent* $N(0, \sigma^2)$ variables.
 - (a) What is the joint pdf $f_{X,Y}(u,v)$ of X and Y ?
 - (b) Sketch the u - v plane and indicate on it the region over which you need to integrate the joint pdf in order to find $P\{X^2 + Y^2 > \alpha^2\}$. Then, compute $P\{X^2 + Y^2 > \alpha^2\}$. Hint: read the Solutions to Problems 4(d) and 6(b) of Problem Set #1 and Problem 1(a) of Problem Set #10. From here onwards, assume $\sigma^2 = 1$ so that X and Y are independent *unit* Gaussian RVs.
 - (c) Express $P\{|X| > \alpha\}$ in terms of the complementary unit Gaussian CDF function $Q(x)$, and use this to write $P\{|X| > \alpha, |Y| > \alpha\}$ in terms of $Q(x)$. (Remember commas mean intersections).
 - (d) Sketch the u - v plane and show on it the region over which you must integrate the joint pdf to find $P\{|X| > \alpha, |Y| > \alpha\}$. Compare the sketches in parts (b) and (d) to deduce that $P\{|X| > \alpha, |Y| > \alpha\} \leq P\{X^2 + Y^2 > 2\alpha^2\}$.
 - (e) Show that the inequality of part (d) implies that $Q(x) \leq (1/2) \cdot \exp(-x^2/2)$ as was proved earlier in Problem 1(d) of Problem Set #10.