University of Illinois | **First Hour Exam** | **ECE 313 Spring 2000**

1. Events A, B and C are defined on a sample space Ω.

(a) If $P(A) = \frac{3}{5}$, $P(A \quad B) = \frac{4}{5}$, and $P(A|B) = \frac{1}{2}$, find P(B).

(b) **Suppose instead** that A and B are *disjoint* events with P(A) = 2P(B), and that $P(C|A) = \frac{3}{7}$ and $P(C|B) = \frac{2}{7}$. Find the value of $P(C|A \quad B)$.

2. The probability that you *will fail* to set your alarm clock tonight to wake you up on time tomorrow morning is $\frac{1}{3}$. If you set the alarm, it will ring on time with probability $\frac{9}{10}$; if you don't set it, it will not ring at all. If the alarm rings, you will wake up on time with probability $\frac{5}{6}$. Even if the alarm does not ring, you will nonetheless somehow manage to wake up on time with probability $\frac{1}{4}$.

What is the probability that you will wake up on time tomorrow?

3. Section A of a course has 30 students of which 10 are excellent, 15 are good, and 5 are average. Section D of the course has 50 students of which 15 are excellent, 20 are good, and 15 are average.

(a) One of these 80 students is picked at random. Let H_0 denote the hypothesis that the student is from Section A and H_1 the hypothesis that the student is from Section D. Complete the *likelihood matrix* shown below by writing the likelihoods of the various observations, and indicate the maximum-likelihood decision rule by shading. Note: the shaded square indicates your choice of hypothesis for the observation.

Likelihood Matrix

	Excellent	**Good**	**Average**
H_0: student is from Section A			
H_1: student is from Section D			

(b) What is the probability that a student from Section A is assumed to be in Section D?
What is the probability that a student from Section D is assumed to be in Section A?
What is the (average) probability of making an incorrect decision?

(c) As in part (a), one of these 80 students is picked at random. Let H_0 denote the hypothesis that the student is from Section A and H_1 the hypothesis that the student is from Section D. Complete the *joint probability matrix* shown below, and indicate the minimum-error-probability decision rule (that is, the Bayesian or maximum *a posteriori* probability decision rule), by shading. Note: the shaded square indicates your choice of hypothesis for the observation.

Joint Probability Matrix

	Excellent	**Good**	**Average**
H_0: student is from Section A			
H_1: student is from Section D			

(d) What is the probability of making an incorrect decision?
Is your answer larger, smaller or the same as the one you found in part (b)?

(e) **(5 points)** Now suppose that **two** students are chosen, one from each section. The choices are independent experiments. We do not know which student belongs to which section, but we observe that one of them is excellent and the other is good. Let H_0 denote the hypothesis that the excellent student is from Section A and the good student is from Section D. Let H_1 denote the hypothesis that the excellent student is from Section D and the good student is from Section A. Which hypothesis is chosen by the maximum-likelihood decision rule in this particular case when one of the students is observed to be excellent and the other is observed to be good?

4. Eight people hold reservations for travel in a 5-passenger limousine from Champaign to St. Louis. The number of persons who actually show up to travel can be modeled as a binomial random variable **X** with parameters $(8, \frac{1}{2})$. Note: $2^8 = 256$.

(a) What is the average value of **X**?

(b) If more than 5 people show up, only the first 5 get to go, and the rest are left behind. What is the average number of passengers who are left behind?