

Assigned: Wednesday, April 14, 1999

Due: Wednesday, April 21, 1999

Reading: Ross, Chapter 6 (except Sections 6.6 and 6.8), Chapter 7 (Sections 1-3 only)

Noncredit Exercises: pp. 293-300: 8-12, 15, 20-23, 26, 28-30; pp. 300-304: 8, 14, 22, 23;

Problems:

1. The discrete random variables $\mathbf{X}$ and $\mathbf{Y}$ have joint pmf $p_{\mathbf{X},\mathbf{Y}}(u,v)$ given by

4	0	1/12	1/6	1/12
3	1/6	1/12	0	1/12
-1	1/12	1/6	1/12	0
$v \uparrow / u \rightarrow$	0	1	3	5

(a) Find the marginal pmfs $p_{\mathbf{X}}(u)$ and $p_{\mathbf{Y}}(v)$ of $\mathbf{X}$ and $\mathbf{Y}$.

(b) Are the random variables $\mathbf{X}$ and $\mathbf{Y}$ independent?

(c) Find $P\{\mathbf{X} \leq \mathbf{Y}\}$ and $P\{\mathbf{X} + \mathbf{Y} \leq 8\}$.

2. The joint pmf of $\mathbf{X}$ and $\mathbf{Y}$ is $p_{\mathbf{X},\mathbf{Y}}(i,j) = 2^{-(i-1)}3^{-j}$, $i = 1, 2, \dots$; $j = 1, 2, \dots$

(a) Find the marginal pmfs of $\mathbf{X}$ and $\mathbf{Y}$ and show that these are valid pmfs. What kinds of random variables are these?

(b) Find the conditional pmf of $\mathbf{X}$ given that $\mathbf{Y} = k$.

(c) Are $\mathbf{X}$ and $\mathbf{Y}$ independent random variables?

3. The number of α -particles emitted by a source during a unit time interval can be modeled as a Poisson random variable $\mathbf{X}$ with parameter λ . The α -particles are detected by means of a (imperfect) Geiger counter which detects a particle with probability $p < 1$. The detections of the various particles can be considered to be independent events. Thus, if n particles have been emitted, the Geiger counter reading can be modeled as a binomial random variable $\mathbf{Y}$ with parameters (n,p) . In short, $p_{\mathbf{Y}|\mathbf{X}}(k|n)$, the *conditional* pmf of $\mathbf{Y}$ given that

$\mathbf{X} = n$, is a binomial pmf: $p_{\mathbf{Y}|\mathbf{X}}(k|n) = \binom{n}{k} p^k (1-p)^{n-k}$ for $0 \leq k \leq n$. (Note that $\mathbf{Y} \leq \mathbf{X}$ always: the counter does not mistakenly count a particle when no particle is present, i.e. there are no false alarms! but the counter does occasionally fail to detect a particle).

(a) Sketch the u - v plane and the joint pmf of $\mathbf{X}$ and $\mathbf{Y}$. Precision is not required in the sizes of the blobs you draw, but be sure that you don't put masses where they do not belong.

(b) What is the unconditional pmf of $\mathbf{Y}$?

(c) What is the conditional pmf of $\mathbf{X}$ given $\mathbf{Y} = k$?

4. The jointly continuous random variables $\mathbf{X}$ and $\mathbf{Y}$ have joint pdf given by

$$f_{\mathbf{X},\mathbf{Y}}(u,v) = \begin{cases} 2 \exp -(u + v), & 0 < u < v < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Sketch the u - v plane and indicate on it the region over which $f_{\mathbf{X},\mathbf{Y}}(u,v)$ is nonzero.

(b) Find the marginal pdfs of $\mathbf{X}$ and $\mathbf{Y}$.

(c) Are the random variables $\mathbf{X}$ and $\mathbf{Y}$ independent?

(d) Find $P\{\mathbf{Y} > 3\mathbf{X}\}$.

(e) For $\alpha > 0$, find $P\{\mathbf{X} + \mathbf{Y} \leq \alpha\}$.

(f) Use the result in (f) to determine the pdf of the random variable $\mathbf{Z} = \mathbf{X} + \mathbf{Y}$.

5. The jointly continuous random variables $\mathbf{X}$ and $\mathbf{Y}$ have joint pdf

$$f_{\mathbf{X},\mathbf{Y}}(u,v) = \begin{cases} 1/2, & 0 \leq u < 1, 0 \leq v < 1, \text{ and } 0 \leq u + v < 1 \\ 3/2, & 0 \leq u < 1, 0 \leq v < 1, \text{ and } 1 \leq u + v < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find $f_{\mathbf{X}}(u)$, $P\{\mathbf{X} + \mathbf{Y} \leq 3/2\}$ and $P\{\mathbf{X}^2 + \mathbf{Y}^2 \geq 1\}$.