ECE 313

SECOND HOUR EXAMINATION

Wednesday April 14, 1999
7:00 p.m. — 8:00 p.m.
269 Everitt Laboratory

Name ________________________________ University Net-ID______________________
Print in BLOCK CAPITALS

Section ☐ C 10 a.m. MWF

☐ D 11 a.m. MWF Signature ____________________________

INSTRUCTIONS

This exam booklet contains four problems on four numbered pages following this cover sheet.

The last page of this exam booklet is a sheet containing a table of $\Phi(x)$, the unit Gaussian CDF, as well as miscellaneous formulas and integrals that may (or may not!) be useful in solving the problems. Feel free to tear off this sheet for ease of reference — the sheet does not have to be turned in with the exam.

This exam is closed book; however, you may consult both sides of your 8.5"×11" sheet of notes. Calculators, laptop computers, Palm Pilots etc. may not be used. Note that the problems are not weighted equally and pace yourself accordingly.

Write your answers in the spaces provided. It is *not necessary* to convert answers that are simple common fractions (such as 10/32) into decimal fractions (10/32 = 0.3125), but you should simplify the fractions by cancelling common factors from the numerator and denominator (thus, write 5/16 instead of 10/32).

Please show all your work. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page.

GRADING

1.	25 points	______
2.	25 points	______
3.	20 points	______
4.	30 points	______
TOTAL	**(100 points)**	______

1. (25 points) **X** is a continuous random variable with pdf

$$f_{\mathbf{X}}(u) = \begin{cases} 0.5\,|\sin(u)|, & -\pi/2 < u < \pi/2, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) (10 points) Find P{**X** > π/4}.

(b) (15 points) If **Y** = cos(**X**), what is the expected value of **Y**?

2. (25 points) Boxes of Cornies, the breakfast of silver medalists, contain a picture of either Luke or Darth with probabilities P(L) = 1/3 and P(D) = 2/3. Little Jimmy Kirk asks his mother to buy cereal boxes until he has acquired **at least one picture of both Luke and Darth.**

Let **X** denote the number of pictures of Luke that Jimmy has at the end of this cereal-eating binge.

(a) (10 points) Find $p_{\mathbf{X}}(u)$, the probability mass function (pmf) of **X**.
For full credit, you must specify the value of $p_{\mathbf{X}}(u)$ for all positive integers u.

(b) (15 points) What is the average value of **X**?

3. (20 points) **X** is a continuous random variable with probability density function

$$f_{\mathbf{X}}(u) = \begin{cases} a + bu, & 0 < u < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

If $E[\mathbf{X}] = \frac{2}{3}$, find $P\{\mathbf{X} < \frac{1}{2}\}$.

If the answer cannot be determined from the given information, check the square □ on the left side of the answer box and leave the right side of the answer box blank.

4. (30 points) **X** denotes a Gaussian random variable with mean 2 and variance 25.

(a) (8 points) Find $P\{|\mathbf{X} - 4| > 3\}$ and $P\{\mathbf{X} < 3 \mid \mathbf{X} > 2\}$ using the table of values of the unit Gaussian CDF Φ(•) on the back of the cover sheet.

(b) (10 points) Let $\mathbf{Y} = \left(\frac{\mathbf{X} - 2}{5}\right)^2$. Write an expression in terms of Φ(•) for $P\{\mathbf{Y} \le v\}$ where v denotes a positive real number.

(c) (12 points) Find the pdf of the random variable **Y** defined in part (b). You must specify the value of $f_{\mathbf{Y}}(v)$ for all real numbers v, $-\infty < v < \infty$.

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(a) (10 points) Find $P\{\mathbf{X} > \pi/4\}$.

(b) (15 points) If $\mathbf{Y} = |\sin(\mathbf{X})|$, what is the expected value of **Y**?

2. (25 points) Boxes of Cornies, the breakfast of silver medalists, contain a picture of either Luke or Darth with probabilities $P(L) = 2/3$ and $P(D) = 1/3$. Little Jimmy Kirk asks his mother to buy cereal boxes until he has acquired **at least one picture of both Luke and Darth.**

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(a) (8 points) Find $P\{|\mathbf{X} - 4| > 3\}$ and $P\{\mathbf{X} < 3 \mid \mathbf{X} > 2\}$ using the table of values of the unit Gaussian CDF $\Phi(\bullet)$.

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(c) (12 points) Find the pdf of the random variable **Y** defined in part (b). You must specify the value of $f_{\mathbf{Y}}(v)$ for all real numbers v, $-\infty < v < \infty$.

$\Phi(x)$, the area under the standard Gaussian density function to the left of x

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

SOME USEFUL RESULTS

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \qquad 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

If $|x| < 1$, then $1 + x + x^2 + x^3 + \ldots = (1-x)^{-1}$ and $1 + 2x + 3x^2 + 4x^3 + \ldots = (1-x)^{-2}$

$$\int u\,dv = uv - \int v\,du \qquad \int \exp(ax)\,dx = a^{-1}\exp(ax) \qquad \int \sin x\,dx = -\cos x \qquad \int \cos x\,dx = \sin x$$

$$\int \frac{dx}{1+x^2} = \arctan x \qquad \int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1}, & \text{if } n \neq -1, \\ \ln x, & \text{if } n = -1. \end{cases} \qquad \int_{-\infty}^{\infty} \exp -(a^2x^2 + 2bx)\,dx = \sqrt{\pi}\,a^{-1}\exp\left(\frac{b^2}{a^2}\right)$$

$$\sin(\pi/6) = \cos(\pi/3) = 1/2, \qquad \sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2 \qquad \sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$$