ECE 313

FIRST HOUR EXAMINATION

Tuesday October 14, 1997
7:00 p.m. — 8:00 p.m.
228 Natural History Building

Name ________________________________ University ID # _______________________
Print in BLOCK CAPITALS

Section ☐ B 9 a.m. MWF

☐ D 11 a.m. MWF Signature ____________________________

INSTRUCTIONS

This exam is closed book; however, you may consult both sides of your 8.5"×11" sheet of notes. Calculators, laptop computers, PDAs etc. may not be used. Note that the problems are not weighted equally and pace yourself accordingly.

Write your answers in the spaces provided. It is *not necessary* to convert answers that are simple common fractions (such as 10/32) into decimal fractions (10/32 = 0.3125), but you should simplify the fractions by cancelling common factors from the numerator and denominator (thus, write 5/16 instead of 10/32).

Please show all your work. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page.

GRADING

1.	25 points	______
2.	20 points	______
3.	30 points	______
4.	25 points	______
TOTAL	**(100 points)**	______

This examination booklet consists of this cover page and four additional numbered pages

1.(a) Events A and B are defined on a sample space.

If $P(A|B) = 2/3$, $P(B|A) = 1/3$, and $P(A \cup B) = 7/8$, find P(B).

If P(B) cannot be determined from the information given, check the square ☐ on the left side of the answer box and do not write anything in the right side of the answer box.

☐ P(B) cannot be determined from the given data.	P(B) =

(b) Events A, B, C, and D are defined on a sample space. If $P(A) = P(B) = 0.1$, $P(C) = 0.5$, and $P(D) = 0.2$, find $P((A \cup B) \cap (C \cup D^c))$ under each of the following two different sets of assumptions:

(i) A, B, C, D are *independent* events

(ii) A, B, C, D are *mutually exclusive* events.

If the probability cannot be calculated from the given data, check the square ☐ on the second line of the answer box and do not write anything on the third line.

A, B, C, D are *independent* events	A, B, C, D are *mutually exclusive* events
☐ The probability cannot be determined	☐ The probability cannot be determined
$P((A \cup B) \cap (C \cup D^c))$ =	$P((A \cup B)\cap(C \cup D^c))$ =

2. (Simplified version of Ross, p. 58, #27: a suggested noncredit exercise)

An urn contains 3 red balls and 3 black balls. A and B take turns drawing balls at random from the urn, with A drawing first. Balls that have been drawn are not replaced in the urn. What is the probability that A draws a red ball before B does?

P(A draws a red ball before B does) =

3. (Ross, p. 56, #13: a suggested noncredit exercise)

A certain town has three newspapers A, B, and C. The proportions of townspeople that read these newspapers are as follows:

A: 10%, B: 30%, C: 5%; A and B: 8%, A and C: 2 %, B and C: 4%,
and 1% read all three newspapers.

(a) What percentage of people read only one newspaper?

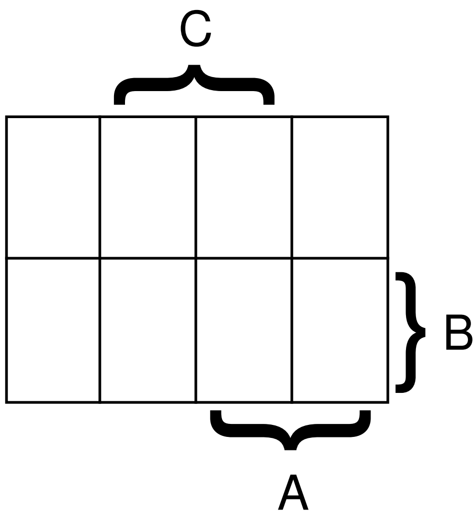
(b) What percentage of people read at least two newspapers?

(c) If A and C are morning papers and B is an evening paper, what percentage of people read at least one morning paper as well as the evening paper?

(d) How many people do not read any newspapers?

_____ % read only one newspaper	_____ % read at least two newspapers
_____ % read at least one morning paper as well as the evening paper	_____ % do not read any newspaper

Use the Karnaugh map/Venn diagram shown below or draw your own.



4. A baseball pitcher can pitch fastballs (event F), curveballs (event C), or sliders (event S). The batter is able to hit fastballs with probability 2/5, curveballs with probability 1/4, and sliders with probability 1/6. It is also known that twice as many pitches are curveballs as are fastballs.

(a) If the batter's batting average is .250 (that is, on average, he succeeds in hitting one of four pitches), what is P(C)?

(b) A fan sitting in the bleachers sees the batter getting a hit. He knows what the batter's chances of hitting each kind of pitch are, but he is sitting too far away to tell whether the pitch was a fastball, a curveball or a slider. What is his maximum-likelihood decision as to what kind of pitch it was?

(c) After cheering the hit, the fan finds that P(F), P(C), and P(S) are listed in the program guide. What is his Bayesian (that is, maximum *a posteriori* probability or minimum-error-probability) decision as to the kind of pitch it was?

P(C) =	Maximum-likelihood decision on seeing a hit: It was a ____________ Bayesian decision on seeing a hit: It was a____________

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Numerical answers depend on which version of the exam you were solving

1.(a) $P(A|B) = \alpha \Rightarrow P(AB) = \alpha P(B)$. $P(B|A) = \beta \Rightarrow P(AB) = \beta P(A)$.
Hence, $P(A) = (\alpha/\beta)P(B)$. Since $P(A \cup B) = P(A) + P(B) - P(AB) = \gamma$, we readily obtain
$(\alpha/\beta)P(B) + P(B) - \alpha P(B) = \gamma$ giving $P(B) = \dfrac{\gamma\beta}{\alpha + \beta - \alpha\beta}$.

(b)(i) If A, B, C, and D are independent, then so are $A \cup B$ and $C \cup D^c$. Hence,
$P((A \cup B) \cap (C \cup D^c)) = P(A \cup B)P(C \cup D^c)$
$= [P(A) + P(B) - P(A)P(B)]\times[P(C) + P(D^c) - P(C)P(D^c)]$

(b)(ii) If A, B, C, and D are mutually exclusive, then
$P((A \cup B) \cap (C \cup D^c)) = P[(A \cup B)\cap C \cup (A \cup B)\cap D^c] = P[(A \cup B)\cap D^c]$ since
$(A \cup B)\cap C = \emptyset$. But, A, B, and C are all subsets of D^c (why?) and hence
$P((A \cup B) \cap (C \cup D^c)) = P[(A \cup B)\cap D^c] = P(A \cup B) = P(A) + P(B)$.

2. A wins if the first ball drawn is red or if the first and second balls are black and the third is red. The probability is thus $\frac{1}{2} + \frac{3}{6}\times\frac{2}{5}\times\frac{3}{4} = \frac{1}{2} + \frac{3}{20} = \frac{13}{20}$.

3. From the given values of $P(ABC) = \alpha$, $P(AB) = \beta$, $P(BC) = \gamma$, and $P(AC) = \delta$, it is easy to find $P(ABC^c) = \beta - \alpha$, $P(A^cBC) = \gamma - \alpha$, and $P(AB^cC) = \delta - \alpha$. Next, since
$P(A) = \varepsilon = P(ABC) + P(ABC^c) + P(AB^cC) + P(AB^cC^c)$, we get $P(AB^cC^c) = \varepsilon - \delta - \beta + \alpha$.
Similarly, if $P(B) = \zeta$ and $P(C) = \eta$, then $P(A^cBC^c) = \zeta - \beta - \gamma + \alpha$, and
$P(A^cB^cC) = \eta - \gamma - \delta + \alpha$. Finally, we can get $P(A \cup B \cup C)$
$= P(ABC) + P(ABC^c) + P(A^cBC) + P(AB^cC) + P(AB^cC^c) + P(A^cBC^c) + P(A^cBC^c)$
$= \varepsilon + \zeta + \eta - (\beta + \gamma + \delta) + \alpha$ or more directly from Proposition 4.4 of Ross. Finally,
$P(A^cB^cC^c) = 1 - [\varepsilon + \zeta + \eta - (\beta + \gamma + \delta) + \alpha]$. The probabilities asked for are

(a) $P(AB^cC^c) + P(A^cBC^c) + P(A^cBC^c) = \varepsilon + \zeta + \eta - 2\beta - 2\gamma - 2\delta + 3\alpha$,

(b) $P(AB \cup BC \cup AC) = P(ABC) + P(ABC^c) + P(A^cBC) + P(AB^cC) = \beta + \gamma + \delta - 2\alpha$,

(c) $P(B \cap (A \cup C)) = \beta + \gamma - \alpha$, and

(d) $P(A^cB^cC^c) = 1 - [\varepsilon + \zeta + \eta - \beta - \gamma - \delta + \alpha]$.

4.(a) $\alpha P(C) = P(F)$ where $\alpha =$ **2** or 1/2 depending on the version of the exam. Also, since the pitcher can only throw fastballs, curveballs or sliders, $P(F) + P(C) + P(S) = 1$. Hence, letting H denote the event that the batter hits the ball, we have that
$0.25 = P(H|C)P(C) + P(H|F)P(F) + P(H|S)P(S)$
$= P(H|C)P(C) + \alpha P(H|F)P(C) + P(H|S)(1 - \alpha P(C) - P(C))$ which can be solved for $P(C)$.
The answer is $P(C) =$ **5/24** or 10/24 = 5/12 depending on the version of the exam

(b) We merely need to compare $P(H|F)$, $P(H|C)$, and $P(H|S)$ to determine which is the largest. The answer is a **curveball** or fastball depending on the version of the exam.

(c) Now compare $P(H|F)P(F)$, $P(H|C)P(C)$, and $P(H|S)P(S)$. Not e that $P(C)$ was found in part (a) and hence we also know $P(F)$ and $P(S)$. In making the comparison, we can express everything on a common denominator of 24 and compare the numerators which are respectively **2.5, 2, and 1.5**, or 2, 2.5, and 1.5 depending on the version of the exam. Thus, the Bayesian decides **fastball** or curveball depending on the version of the exam.