University of Illinois **Fall 2026**

ECE 313: Conflict Exam I

Tuesday, October 13, 2026
8:00 a.m. — 9:30 a.m.

Name: (in BLOCK CAPITALS) _____

NetID: _____

Signature: _____

Section: ☐ A, MWF at 2 (Katselis) ☐ B, MWF at 10 (Zhao) ☐ C, MWF at 11 (Katselis) ☐ D, MWF at 1 (Katselis)

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, microscopes, etc. are not allowed.

The exam consists of four problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. **Draw a small box around each of your final numerical answers.**

Grading	
1. 20 points	_____
2. 20 points	_____
3. 30 points	_____
4. 30 points	_____
Total (100 points)	_____

1. **[20 points]**

Suppose that a fair coin is independently tossed four times. Define the following events:

$$A = \{\text{The outcomes of tosses 1 and 3 are heads}\},$$
$$B = \{\text{The outcomes of tosses 2 and 3 are heads}\},$$
$$C = \{\text{The outcomes of tosses 1, 2, and 3 are the same}\}.$$

(a) (10 pts) Are A, B, and C pairwise independent? Justify your answer.

(b) (10 pts) Are A, B, and C mutually independent? Justify your answer.

2. **[20 points]**

(a) (10 pts) Let $X \sim \text{Geo}(1/2)$. Find $\text{Var}(3X + 1) + \mathbb{E}[X^2 + 2X]$.

(b) (10 pts) Four fair six-sided dice are rolled independently. Let X be the total number of dice showing a 5. Find $P(X \geq 1 \mid X \leq 3)$.

3. **[30 points]**

Let X and Y be two nonnegative random variables such that

$$\mathbb{E}[X] = 2, \qquad \mathbb{E}[Y] = 4, \qquad \mathrm{Var}(X) = 1, \qquad \mathrm{Var}(Y) = 2.$$

(a) (8 pts) Upper bound $P(X + Y \geq 10)$ using Markov's inequality.

(b) (8 pts) Provide an upper bound for $P(X^2 + Y^2 \geq 50)$.

(c) (8 pts) Upper bound $P(X \geq 4)$ using Chebyshev's inequality.

(d) (6 pts) **True or False**: $P(Y < 6) \geq \frac{1}{3}$. Justify your answer.

4. **[30 points]**

Consider a binary hypothesis testing problem with the following two hypotheses:

$$H_0 : X \sim \text{Geo}(1/2), \qquad H_1 : X \sim \text{Pois}(1).$$

(a) (7 pts) Find the maximum likelihood (ML) decision rule.

(b) (6 pts) Find the false alarm probability $p_{\text{false alarm}}$ and the miss probability p_{miss} for the ML decision rule.

(c) (7 pts) Find the maximum a posteriori (MAP) decision rule, assuming $\pi_0 = 4\pi_1$.

(d) (10 pts) Find the average probability of error, p_e, for both the ML and MAP decision rules using the prior probabilities given in part (c).