University of Illinois **Fall 2026**

ECE 313: Problem Set 1

Due: Friday, September 11 at 07:00:00 p.m.
Reading: *ECE 313 Course Notes,* Chapter 1

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.
Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. **[Sample Space]**
   Consider events A and B defined on a sample space. If the probability that at least one of the two events occurred is 0.7 and the probability that at least one of the events did *not* occur is 0.9, what is the probability that *exactly one* of the two events occurred?

2. **[Bits]**
   An experiment consists of observing the contents of an 8-bit register. We assume that all 256 byte values are equally likely to be observed.

   (a) Let A denote the event that the least significant bit is a ONE. What is $P(A)$?

   (b) Let B denote the event that the register contains 5 ONEs and 3 ZEROs. What is $P(B)$?

   (c) What is $P(A \cup B)$? What is $P(A \cap B)$? What is the probability that exactly one of A and B occur, i.e. what is $P(A \oplus B)$?

   (d) Let C denote the event that there are 4 ONEs and 4 ZEROs in the register. What is $P(C)$?

3. **[Subsets]**
   Find $P(A \cup (B^c \cup C^c)^c)$ in each of the following four cases:

   (a) A, B, and C are disjoint (mutually exclusive) events and $P(A) = 1/3$.

   (b) $P(A) = 2P(B \cap C) = 4P(A \cap B \cap C) = 1/2$.

   (c) $P(A) = 1/2$, $P(B \cap C) = 1/3$, and $P(A \cap C) = 0$.

   (d) $P(A^c \cap (B^c \cup C^c)) = 0.6$.

4. **[Cards]**
Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3, with all possibilities being equally likely.

   (a) $E1$ is the event that all five cards are one of A, K, Q, J (though not necessarily of the same suit). Find $P(E1)$.

   (b) $E2$ is the event that either all the five cards are from the same suit or four of the five cards are A, K, Q, J from a common suit. Find $P(E2)$.

5. **[Counting rectangles on a chessboard]**
Consider a 6x6 chessboard, which consists of 36 squares in 6 rows and 6 columns.

   (a) How many different rectangles, comprised entirely of chessboard squares, can be drawn on the chessboard? Hint: there are 7 horizontal and 7 vertical lines in the chessboard.

   (b) One of the rectangles you counted in part (a) is chosen at random. What is the probability that it is square shaped?

6. **[First and second moments of a ternary random variable]**
This problem focuses on the possible mean and variance of a random variable X with support set $\{-1, 0, 1\}$. Let $p_X(-1) = a$ and $p_X(1) = b$ and $p_X(0) = 1 - a - b$. Let $\mu = E[X]$, $\sigma^2 = \text{Var}(X)$, and $m_2 = E[X^2]$.

   (a) What are the conditions for a and b to make the pmf valid?

   (b) Find (a, b) so that $\mu = \frac{1}{2}$ and $\sigma = \frac{1}{2}$.

   (c) Express a and b in terms of μ and m_2. Determine and sketch the region of (μ, m_2) pairs for which there is a valid choice of (a, b).

   (d) Determine and sketch the set of (μ, σ^2) pairs for which there is a valid choice of (a, b).