Last lecture

Joint Gaussian Distribution (Ch 4.11)

- Motivation
- Facts
- Examples

Agenda

Final Remarks about the course

- Where can I apply probabilities?
- Correlated advanced courses
- Probability mindset

Final Review I (Pre-midterm2)

- Discrete and continuous Distributions
- Poisson distribution/ process
- Gaussian and Φ / Q functions

Final Remarks

Where can I apply probability?

- ML – VAE, Diffusion, Bayesian DL, RL
    - Law of Large Number is everywhere in data. Noise is inherent

- Communication networks – Queueing, error coding, fading
    - Every bits is a Bernoulli

- Signal Processing – Kalman filter, Wiener filer, spectral analysis
    - Noise is the signal's shadow — learn the shadow, and you master the signal
- Robotics & control – Tracking, SLAM, sensor fusion
    - Model the unknowns – That's reality

Where can I apply probability?

- Computer Security – Cryptography strength, side channel attacks
  - Security is fundamentally a probability game

- Semiconductor Devices – Reliability & lifetime analysis
  - At microscopic scales, randomness is the rule, not the exception

What advanced courses are correlated

- ECE 418 — Image & Video Processing
- ECE 420 / 551 — DSP and Advanced DSP
- ECE 438 — Communication Networks
- ECE 434 — Real-World Algorithms for IoT and Data Science
- ECE 448/449/494 — AI/ ML
- ECE 486 / 489 / 515 — Control Systems
- ECE 498/598RR — Deep Generative Models
- ECE 534 — Random Process
- ECE 543 — Statistical Learning Theory
- ECE 561 — Statistical Inference for Engineers and Data Scientists
- And more...

Probability mindset

- What is the value? → What is the distribution?
  - Always assume a RV, collapse to constant when $\sigma \approx 0$
- Uncertainty is a chance, not a nuisance
  - Model Gaussian, and focus on None-Gaussian
- What distribution would generate observation X?
  - Diffusion, Kalman, Markov model, Likelihood
- When in doubt: model uncertainty explicitly
  - What varies? How does it affect the output

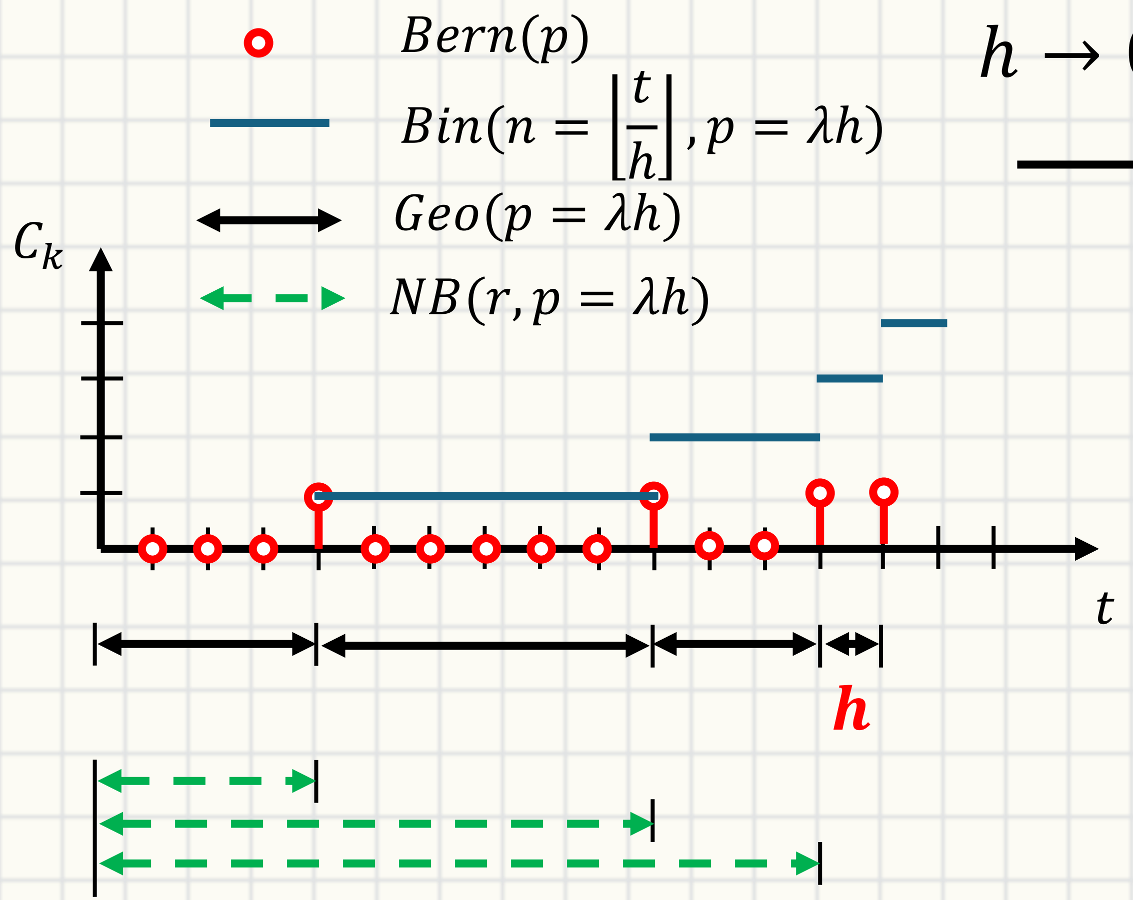
Closing Messages

- Probability isn’t a chapter in the textbook—it’s the operating system of the real world.
- If there is one habit to carry forward from ECE 313, let it be: Whenever something looks uncertain, irregular, or ‘noisy’—don’t guess, don’t simplify—model it.
- Great engineers don’t eliminate randomness; they understand it, quantify it, and use it.

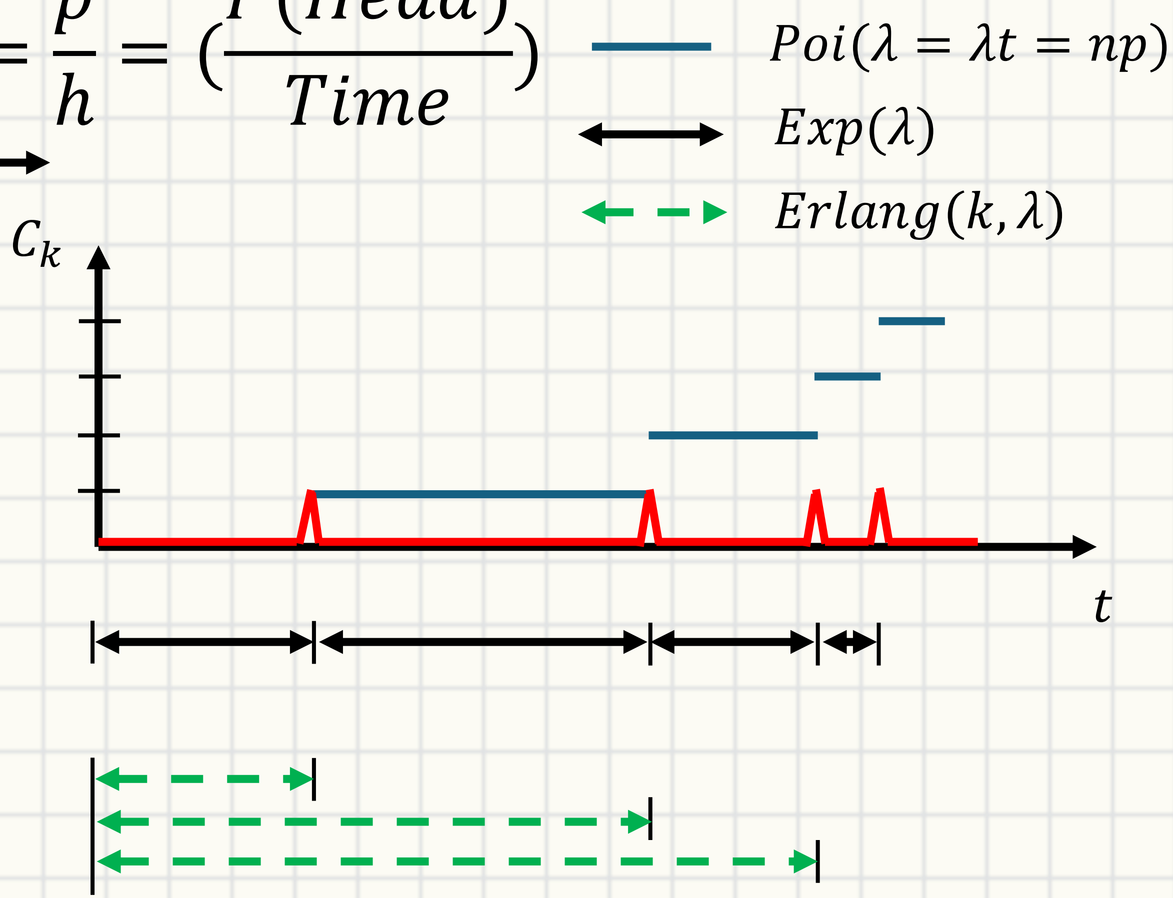
Final Review I (Pre-midterm2)

Bernoulli Process $\xrightarrow{h \to 0, \lambda = \frac{p}{h}}$ Poisson Process

- Assume each trial takes h duration to complete



$$\xrightarrow{h \to 0, \lambda = \frac{p}{h} = (\frac{P(Head)}{Time})}$$



Properties

	$Bern(p)$	$Bin(n,p)$	$Poi(\lambda = np)$	$Geo(p)$	$NB(r,p)$
Def	X_i	$\sum_n X_i$	$\sum_n X_i, n \to \infty$	Y_i	$\sum_n Y_i$
μ	p	np	λ	$1/p$	r/p
σ^2	$p(1-p)$	$np(1-p)$	λ	$\frac{1-p}{p^2}$	$\frac{r(1-p)}{p^2}$
$f_X/p_X(k)$	p or $1-p$	$\binom{n}{k} p^k (1-p)^{n-k}$	$\frac{\lambda^k e^{-\lambda}}{k!}$	$(1-p)^{k-1}p$	$\binom{k-1}{r-1}(1-p)^{k-r}p^r$
Special		$(p+q)^n$	$Poi(np) \approx Bin(n,p)$	Memoryless	

Coin Toss

Toss N coin

Bo K.

$\lambda = np$

Toss until

Toss until rth

Properties

	$Exp(\lambda)$	$Poi(\lambda = \lambda t)$	$Uni[a,b]$	$N(\mu,\sigma^2)$
Def	T_i	$\sum_n X_i, n \to \infty$		
μ	$1/\lambda$	λ	$\frac{a+b}{2}$	μ
σ^2	$1/\lambda^2$	λ	$\frac{(b-a)^2}{12}$	σ^2
$f_X/p_X(k)$	$\lambda e^{-\lambda t}$	$\frac{\lambda^k e^{-\lambda}}{k!}$	$\frac{1}{b-a}$	$\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(x-\mu)^2}{2\sigma^2})$
Special	Memoryless			$F_N(x) \triangleq \Phi(x)$

Fail / continuous time

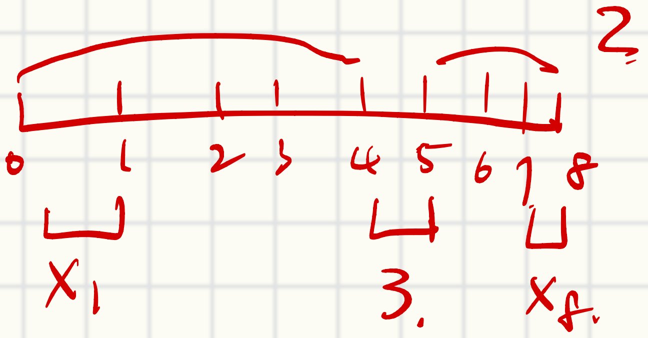
Count # of fails.

Σ Many Binomial $\to N$

Examples – Poisson Process

- Let N_t denotes a Poisson process of rate λ

$X_n \sim Poi(\lambda)$



- $P\{N_8 = 5 | N_5 - N_4 = 3\}$

$N_t = \sum_{n=1}^{t} X_n$

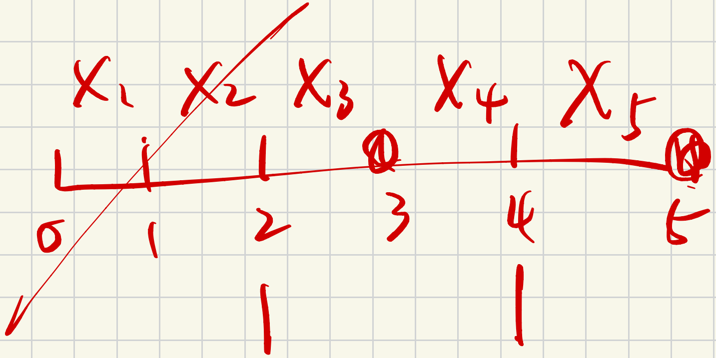
- $P\{\underbrace{N_5 - N_3 = 3}_{W} | \underbrace{N_4 - N_2 = 1}_{Z}\}$

$= \frac{P\{W Z\}}{P\{Z\}}$

$X_1 + X_2 \ldots X_4 + X_6 + X_7 + X_8 = 2$

$\sim Poi(7\lambda)$

$P = \frac{(7\lambda)^2 e^{-7\lambda}}{2!}$



$X_3 + X_4 = 1$

$X_4 + X_5 = 3 \mid X_3 + X_4 = 1$

$X_i \sim Poi(\lambda)$

$P\{X_3 = 1 \quad X_4 = 0 \quad X_5 = 3\}$

$P\{N_4 - N_2 = 1\} = \frac{(2\lambda)^1 e^{-2\lambda}}{1!} + P\{X_3 = 0 \quad X_4 = 1 \quad X_5 = 2$

B

$$\frac{\lambda e^{-\lambda}}{1!} \times e^{-\lambda} \times \frac{\lambda^3 e^{-\lambda}}{3!}$$

$$+)\ e^{-\lambda} \times \frac{\lambda e^{-\lambda}}{1!} \times \frac{\lambda^2 e^{-\lambda}}{2!}$$

S

$$P = \frac{S}{B}$$

Gaussian – Standard Normal

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})$$

$$\frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2})$$

$Z \sim N(0,1)$

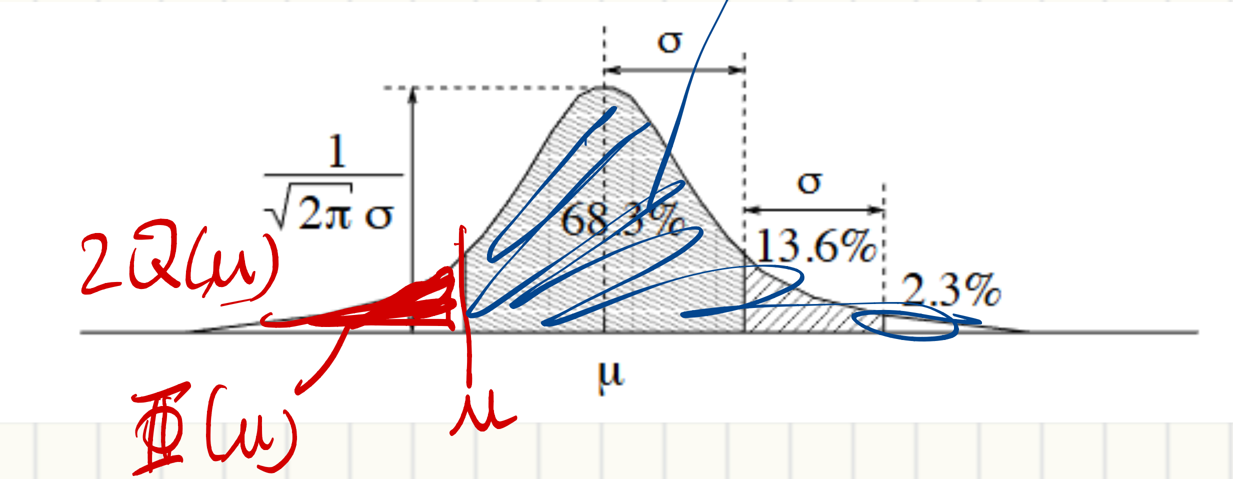
- $\Phi(u) \triangleq F_Z(u) = \int_{-\infty}^{u} f_Z(x)\,du$
- $Q(u) = 1 - \Phi(u)$

CCDF

$P\{|Z| \geq u\}$ $\quad u>0$

$= Q(u) + \Phi(-u) =$

$Q(u)$



Gaussian – General Gaussian

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$$

$X \sim N(\mu, \sigma^2)$

$$Z = \frac{X-\mu}{\sigma}$$

- $F_X(x) =$

$$P\{X \leq x\}$$

$$= P\left\{\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right\} = P\left\{Z \leq \frac{x-\mu}{\sigma}\right\}$$

$$= \Phi\left(\frac{x-\mu}{\sigma}\right)$$