Last lecture

Independent RV (Ch 4.4)

- Examples

Sum of joint RVs (Ch 4.5)

- Motivation
- Discrete RVs & Examples

Agenda

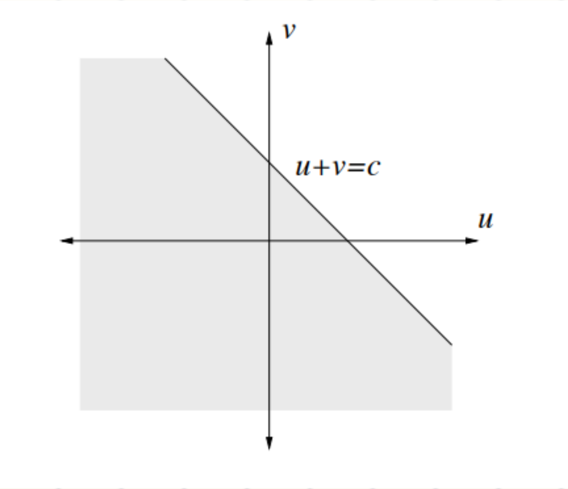
Sum of joint RVs (Ch 4.5)

- Continuous RVs & Examples
- Sum of Gaussians

More examples on joint RVs (Ch 4.6)

- Max of two RVs
- Buffon's needle problems
- Maximum likelihood estimator

Sums of Continuous RVs



Let $S = X + Y$

- $F_S(c) = P\{S \leq c\} =$

- $f_S(c) = \frac{dF_S(c)}{dc} =$

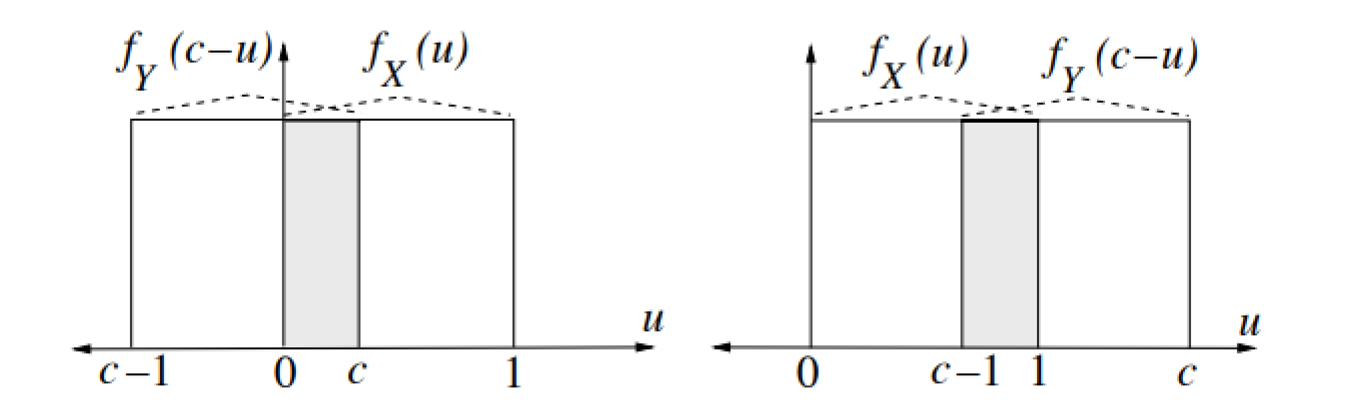
If X and Y are independent

- $f_S(c) =$

Examples

Suppose X and Y are independent, $X, Y \sim Uniform[0, 1]$. Find the pdf of $S = X + Y$

- $f_S = f_X * f_Y$
- What is $f_Y(c - u)$?



- If $0 < c \leq 1,\ f_X * f_Y =$
- If $1 < c \leq 2,\ f_X * f_Y =$

Notes on Gaussian

Assume $X \sim N(0, \sigma_1^2)$, $Y \sim N(0, \sigma_1^2)$

- Sum of two Gaussian of same mean
  - Mean keeps the same
  - $\sigma_S^2 = \sigma_X^2 + \sigma_Y^2$

  - Tedious proof in textbook formula (4.20)
  - But high-level idea - approximate by Binomials…

More examples on joint RVs

Max of two RVs

Let $W = \max(X, Y)$

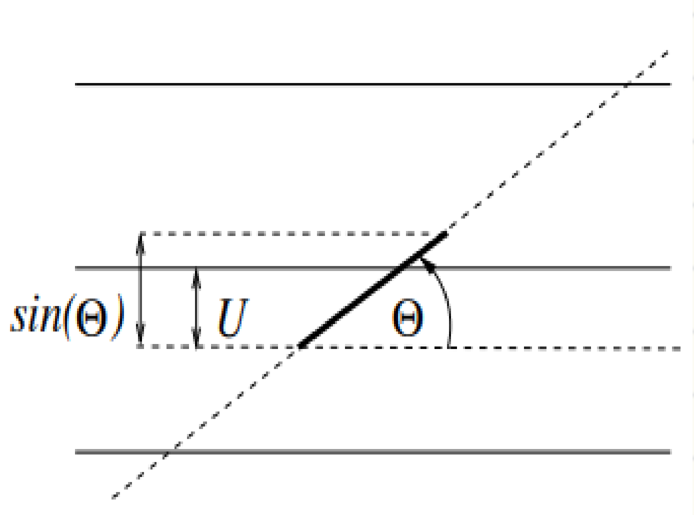
- $F_W(t) = P\{W \leq t\} =$

- $f_W(t) = \frac{dF_W(t)}{dt} =$

Abstract – on $P\{W \in (t, t+h]\} = f_W(t)h + o(h)$

- Case (a): $Y \leq t, X \in (t, t+h]$
- Case (b): $X \leq t, Y \in (t, t+h]$
- Case (c): $X \in (t, t+h], Y \in (t, t+h]$

Buffon's needle problem

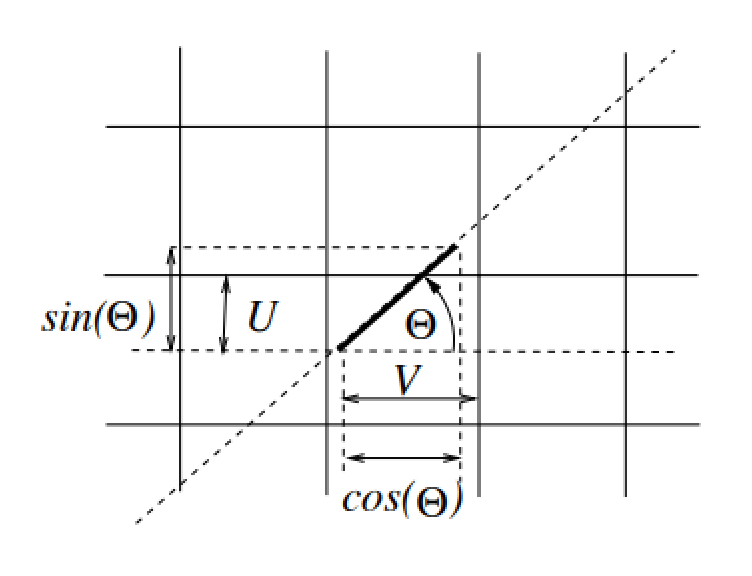


- Draw many parallel horizontal lines
  - Space 1 inch between two lines
  - Throw a needle of 1 inch length on the plane
  - Find $P\{$ "The needle intersect with a line" $\}$

Define U = "distance from the needle lower end to the first line above

Buffon’s needle problem (2)

- What if there are “horizontal” and “vertical” lines?



Let M_h denotes “missing horizontal lines”
M_v denotes “missing vertical lines”

Maximum Likelihood Estimator

A drone is accelerating constantly with unknown rate

- At time t, the location is bt
- Measurement $X_t = bt + W_t$
  - $W_t \sim N(0,1)$ is the independent random noise
- Given $X_{1:T} = u_{1:T}$ as the observation, find $\hat{b}_{ML}$
- Is $\hat{b}_{ML}$ **unbiased**, i.e., $\hat{b}_{ML}(\mu_{1:T}) = b$