

Last lecture

Joint PDF (Ch 4.3)

- Example
 - Uniform distribution
 - Conditional distribution

Independent RV (Ch 4.4)

- From event to RV - CDF
- Check using PDF

Agenda

Independent RV (Ch 4.4)

- Examples

Product Set

Let A, B denote a finite union of intervals

- $|A|$ denotes the total length of A

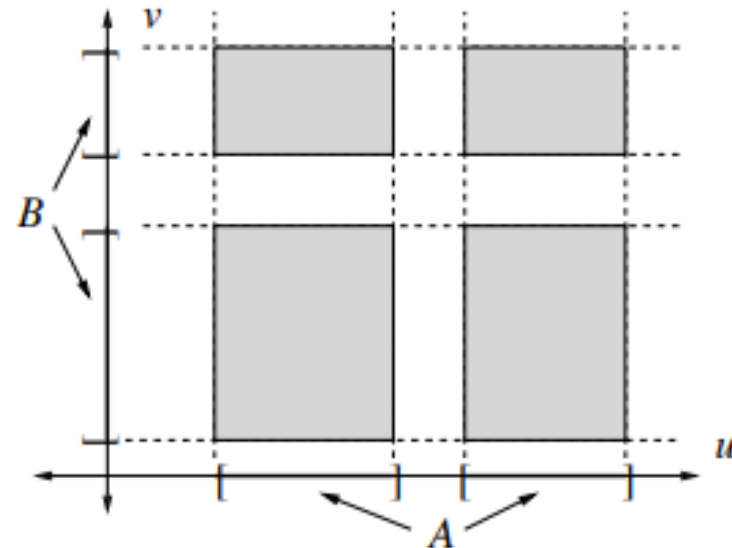
The **product set** $A \times B = \{(u, v) : u \in A, v \in B\}$

- The total area $|A \times B| = |A| \times |B|$

Swap property: $S \in \mathbb{R}^2$ has the **swap property** if

- For any pair of points $(a, b), (c, d) \in S$, (a, d) and (c, b) also in S

Proposition - $S \in \mathbb{R}^2$, S is a product set if and only if it has the swap property



Properties of independent

- If X, Y are independent and jointly continuous type RVs, then support of $f_{X,Y}$ is a product set
- Support X, Y are uniformly distributed over set $S \in \mathbb{R}^2$, then X and Y are independent iif S is a product set

Examples

Decide whether the if X and Y are independent if

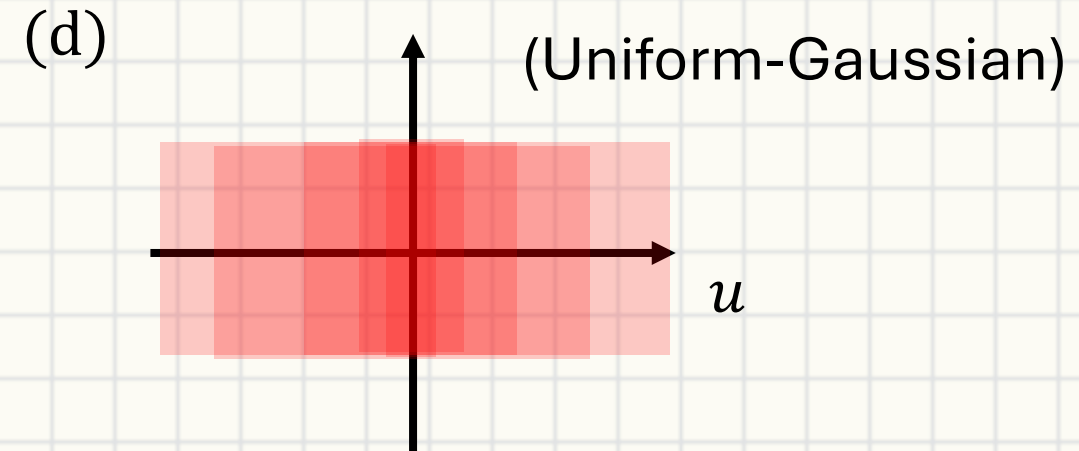
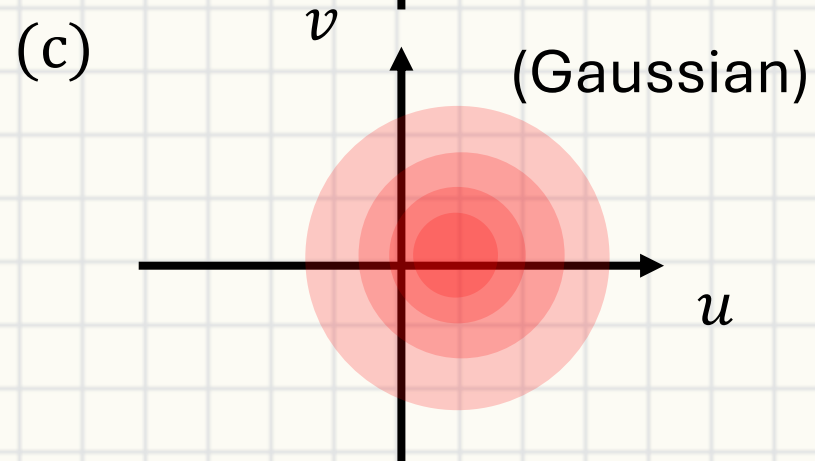
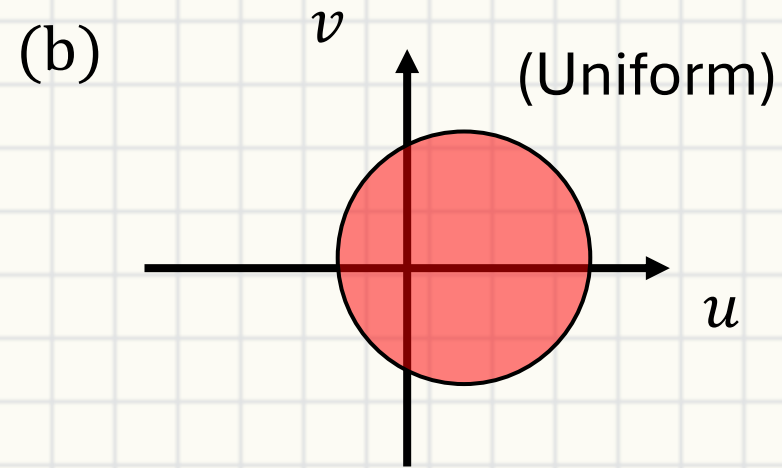
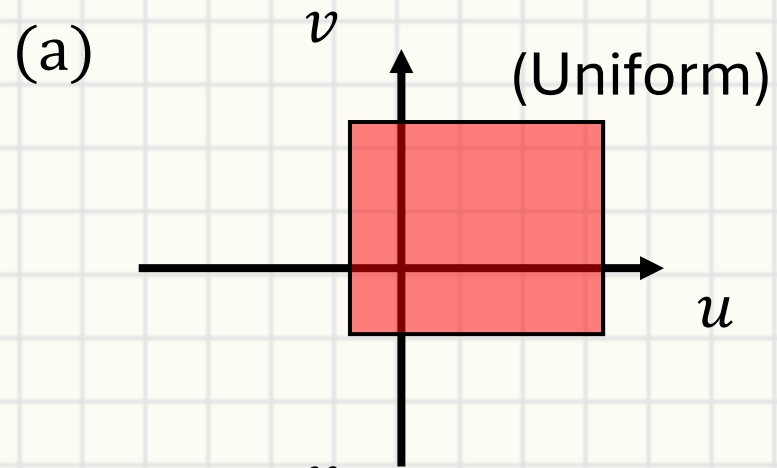
- $f_{X,Y}(u, v) = Cu^2v^2$ for $u, v > 0$ and $u + v \leq 1$; 0 else
- $f_{X,Y}(u, v) = u + v$ for $u, v \in [0,1]$; 0 else
- $f_{X,Y}(u, v) = 9u^2v^2$ for $u, v \in [0,1]$; 0 else

Slido

X and Y are independent based if $f_{XY}(u, v)$ is...



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Sums of joint RVs

Motivation

Recall, we learnt if X and Y are independent

- $E[X + Y] = E[X] + E[Y]$
- $\sigma_{X+Y} = \sigma_X + \sigma_Y$

What if we know p_{XY} or f_{XY} ?

- $E[X + Y] = E[X] + E[Y]$ still holds
- What's the sum of your midterm #1 and midterm #2

Sums of Discrete RVs

Let $S = X + Y$

- $p_S(k) =$

If X and Y are independent, $p_{XY}(j, k - j) = p_X(j)p_Y(k - j)$

- $p_S(k) =$

- Denoted as

Example

Let $X = Bi(n, p)$ and $Y = Bi(m, p)$. $S = X + Y$. Find $p_S(k)$ if X and Y are independent

- Intuitively, $X + Y$ equals “toss a p Head coin $n + m$ times”
- Verify with formula
- $p_S(k) = \sum_{j=0}^k p_X(j)p_Y(k-j)$

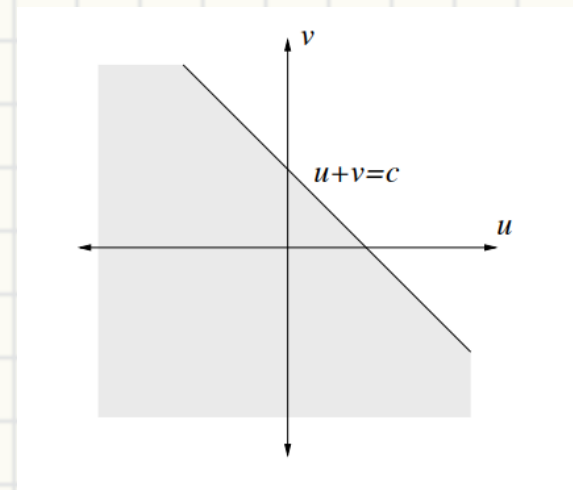
Sums of Continuous RVs

Let $S = X + Y$

- $F_S(c) = P\{S \leq c\} =$
- $f_S(c) = \frac{dF_S(c)}{dc} =$

If X and Y are independent

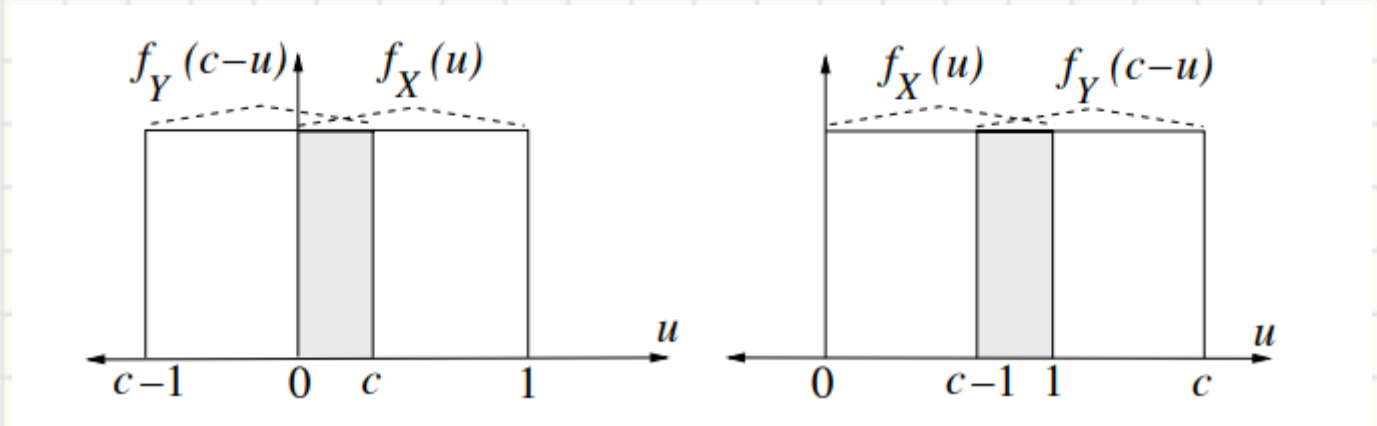
- $f_S(c) =$



Examples

Suppose X and Y are independent, $X, Y \sim \text{Uniform}[0, 1]$. Find the pdf of $S = X + Y$

- $f_S = f_X * f_Y$
- What is $f_Y(c - u)$?



- If $0 < c \leq 1$, $f_X * f_Y =$
- If $1 < c \leq 2$, $f_X * f_Y =$