Last lecture

Binary Hypothesis Testing with continuous distribution (Ch 3.10)

- Example

Jointly Distributed RV/ Joint CDF (Ch 4.1)

- Motivation
- Definition
- Properties

Agenda

Joint PMF (Ch 4.2)

- Definition
- Example

Joint PDF (Ch 4.2)

- Definition
- Example

Joint PMF

If X and Y are discrete, joint PMF $p_{X,Y}(u,v) = P\{X = u, Y = v\}$

Marginalization

- Getting single RV PMF/ PDF from joint PMF/ PDF
- $p_X(u) = \sum_{v_j} p_{X,Y}(u, v_j)$ called “marginal PMF”

Conditional PMF

- $p_{Y|X}(v|u_0) = \frac{p_{X,Y}(u_0,v)}{p_X(u_0)}$

Example

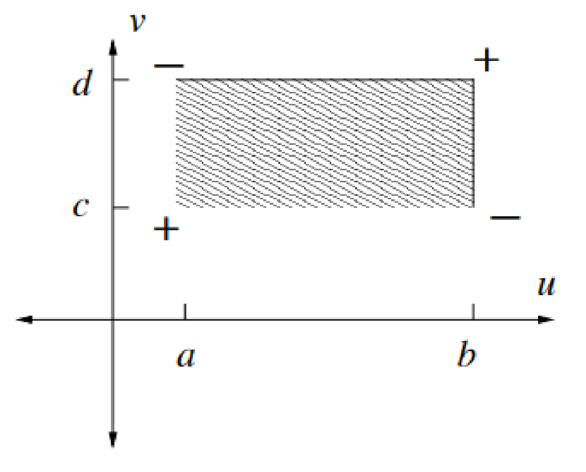
Given joint PMF $p_{X,Y}$ as the table, find

- p_X
- p_Y
- $P\{X = Y\}$
- $P\{X > Y\}$
- $p_{Y|X}(v|2)$

$Y = 3$	0.1	0.1	
$Y = 2$		0.2	0.2
$Y = 1$		0.3	0.1
	$X = 1$	$X = 2$	$X = 3$

Joint PDF

Joint PDF



If X and Y are continuous, joint PDF $f_{X,Y}(u,v)$ s.t.

- $F_{X,Y}(u,v) = \int_{-\infty}^{u} \int_{-\infty}^{v} f_{X,Y}(u,v)dvdu$
- $P\{(X,Y) \in R\} = \int \int_R f_{X,Y}(u,v)dudv$ for piecewise diff. R
- $f_{X,Y}(u,v) \geq 0$ for any $(u,v) \in \mathbb{R}^2$
- $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(u,v)dudv =$

LOTUS: $E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(u,v) f_{X,Y}(u,v)dudv$

- $E[X] =$

Joint PDF

$$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(u,v) f_{X,Y}(u,v)\,du\,dv$$

- $E[aX + bY + c] =$

Marginalization - Projection

- $f_X(u) = \int_{-\infty}^{\infty} f_{X,Y}(u,v)\,dv$
- $E[X] = \int_{-\infty}^{\infty} f_{X,Y}(u,v)\,dv$

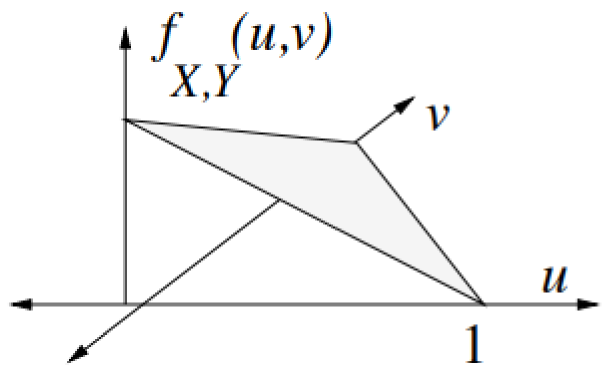
Conditional PDF - Slice

- $f_{Y|X}(v|u_0) = \frac{f_{X,Y}(u_0,v)}{f_X(u_0)}$
- Conditional expectation $E[Y|X = u_0] = \int_{-\infty}^{\infty} v f_{Y|X}(v|u_0)\,dv$

Example

Given $f_{X,Y}(u,v) = \begin{cases} c(1-u-v) & if\ u,v \geq 0, u+v \leq 1 \\ 0 & else \end{cases}$, solve

- c
- Marginal PDF $f_X(u)$
- Conditional PDF $f_{Y|X}(v|u_0)$



Uniform joint PDF

If pdf are constant over support S

- $f_{X,Y}(u,v) = \begin{cases} \frac{1}{area(S)} & if\ (u,v) \in S \\ 0 & else \end{cases}$
- S can be none-rectangular
- $P\{(X,Y) \in A\} = \frac{area(A \cap S)}{area(S)}$

Example

Let (X, Y) uniformly distributed over the uniform disk.

- $f_{X,Y}(u,v) = \begin{cases} c & if\ u^2 + v^2 \leq 1 \\ 0 & else \end{cases}$, solve
- c
- $P\{X \geq 0 \cap Y \geq 0\}$
- $P\{X^2 + Y^2 \leq r^2\}$
- $f_X(u)$
- $f_{Y|X}(v|u_0)$