Last lecture

Generating a customized RV (Ch 3.8.2)

- Intuition $g = F_X^{-1}$
- Examples
  - Uniform to Exponential
  - Uniform to D6 outcome
- Area rule – Compute $E[X]$ using F_X (Ch 3.8.3)

Binary Hypothesis Testing with continuous distribution (Ch 3.10)

- Overview

Agenda

Binary Hypothesis Testing with continuous distribution (Ch 3.10)

- Example

Jointly Distributed RV/ Joint CDF (Ch 4.1)

- Motivation
- Definition
- Properties

Joint PMF (Ch 4.2)

- Definition
- Example

Binary Hypothesis Testing on Continuous Distribution

Overview

Similar to discrete, but with some changes

- $P\{X = u|H_1\} \rightarrow f_1(u)$
- Likelihood Ratio $\Lambda(u) = \dfrac{f_1(u)}{f_2(u)}$
- LRT rule $\Lambda(X) \begin{cases} > \tau & H_1 \\ < \tau & H_0 \end{cases}$

$p_{false\ alarm}, p_{miss}, p_e$ remain the same

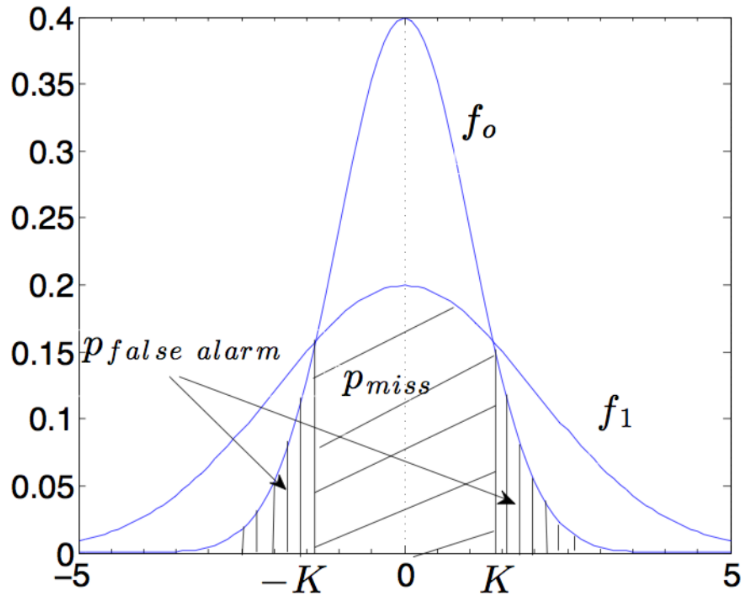
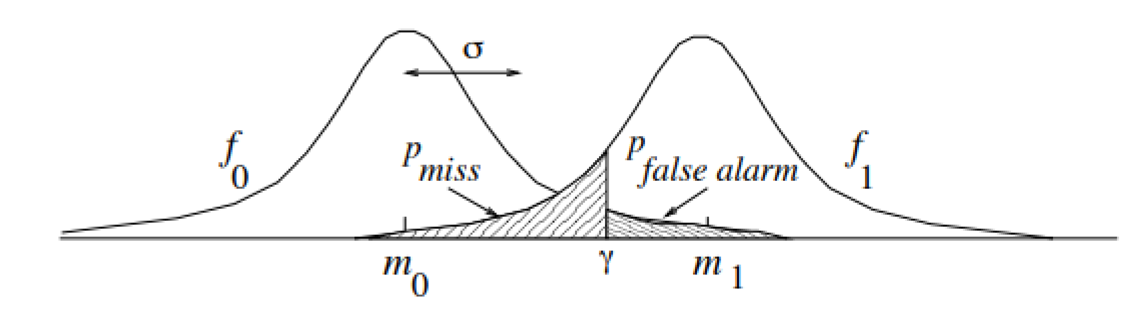


Figure 3.27: $N(0,1)$ and $N(0,4)$ pdfs and ML threshold K.

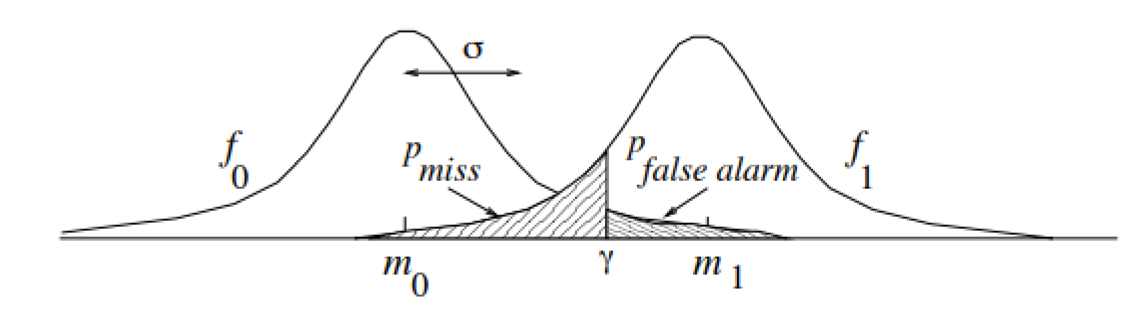
Example



X under H_i follows $N(m_i, \sigma^2)$. Given m_i, σ, π_i, Find ML and MAP rule

- $f_i(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(u-m_i)^2}{2\sigma^2}\right\}$
- $\Lambda(u) = \exp\{(u - \frac{m_0+m_1}{2})(\frac{m_1-m_0}{\sigma^2})\}$
- ML rule –

Example



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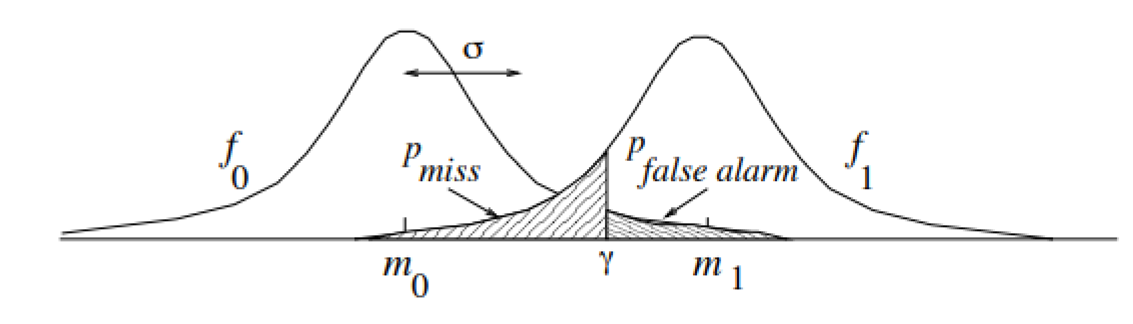
- MAP rule –

Example



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- $P_{miss} =$
- $P_{false\ alarm} =$

Jointly Distributed Random Variables

Motivation

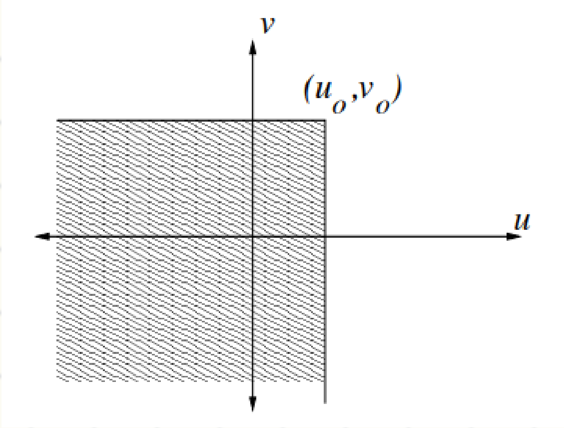
Given X and Y, we have learnt

- Independence
- Function & Scaling (e.g. X=3Y-2)

But real-world cases are more complex

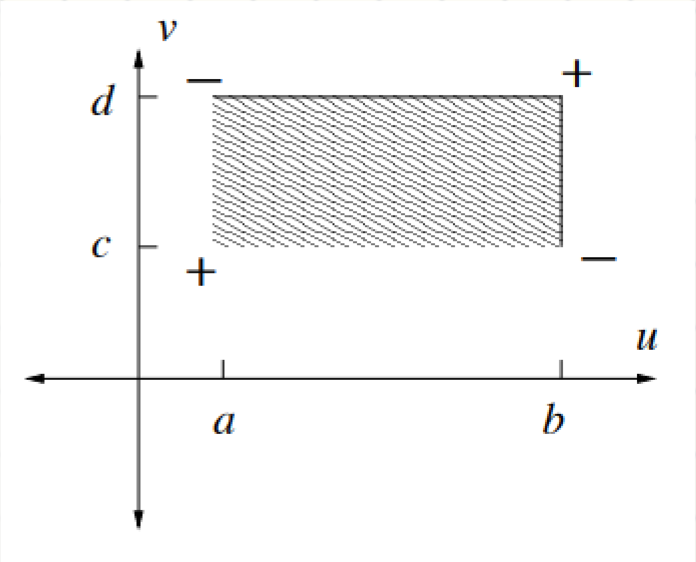
- How to show the "   " between X and Y
- "Jointly distributed" RVs
- $F_X(u) \quad \rightarrow \quad F_{X,Y}(u, v)$
- $P_X(u) \quad \rightarrow \quad P_{X,Y}(u, v)$

Joint CDF



Joint CDF of X and Y

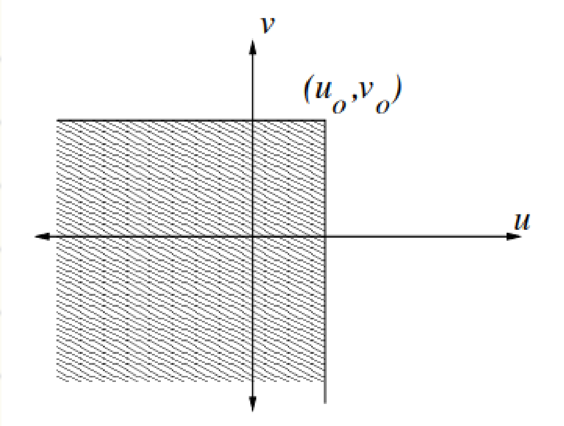
- $F_{X,Y}(u,v) = P\{X < u, Y < v\}$ for any $(u,v) \in \mathbb{R}^2$
- Completely defines all events concerning X and Y
- For a 2D rectangle region $R = (a,b] \times (c,d]$
  - $P\{(X,Y) \in R\} =$
- $F_X(u) = F_{X,Y}(u,\infty)$

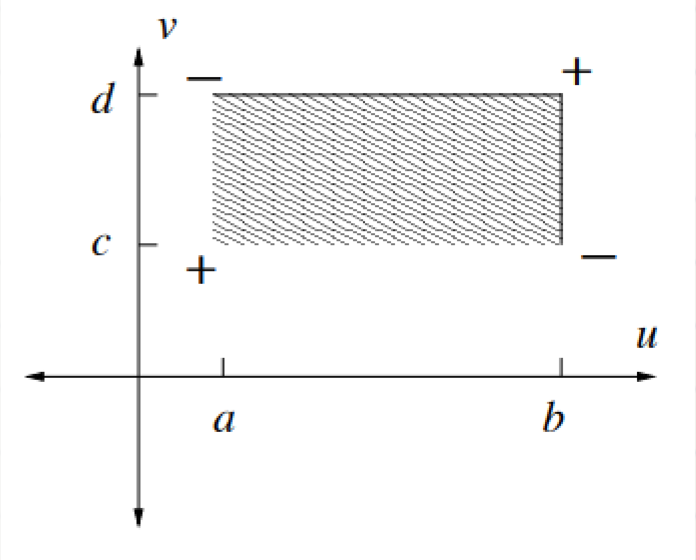


Joint CDF Properties

Denote $F_{X,Y}$ as F

- $0 \leq F(u,v) \leq 1$ for all $(u,v) \in \mathbb{R}^2$
- along u and along v respectively
  - F is none decreasing
  - F is right-continuous
- $\lim_{u\to-\infty} F(u,v) = 0$
- $\lim\limits_{u\to\infty}\lim\limits_{v\to\infty} F(u,v) = 0$





Joint PMF

If X and Y are discrete, joint PMF $p_{X,Y}(u,v) = P\{X = u, Y = v\}$

Marginalization

- Getting single RV PMF/ PDF from joint PMF/ PDF
- $p_X(u) = \sum_{v_j} p_{X,Y}(u, v_j)$ called “marginal PMF”

Conditional PMF

- $p_{Y|X}(v|u_0) = \frac{p_{X,Y}(u_0,v)}{p_X(u_0)}$

Example

Given joint PMF $p_{X,Y}$ as the table, find

$Y=3$	0.1	0.1	
$Y=2$		0.2	0.2
$Y=1$		0.3	0.1
	$X=1$	$X=2$	$X=3$

- p_X
- p_Y
- $P\{X = Y\}$
- $P\{X > Y\}$
- $p_{Y|X}(v|2)$