ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

LECTURE 4 : COUNTING AND PROBABILITY EXPERIMENTS WITH EQUALLY LIKELY OUTCOMES

- TOPICS TO COVER (BASED ON CH 1.3 - 1.4)

→ CALCULATING THE SIZE OF VARIOUS SETS

→ PROBABILITY EXPERIMENTS WITH EQUALLY LIKELY OUTCOMES

→ CALCULATING THE SIZE OF VARIOUS SETS

Ω : FINITE

ALL OUTCOMES ARE EQUALLY LIKELY

$A \in F$: $P(A) = \frac{|A|}{|\Omega|}$ $|\cdot|$: CARDINALITY

E.G. $\Omega = \{H, T\}$ ASSUME $P(\{H\}) = \frac{1}{2} = P(\{T\})$

$A \in F$: $P(A) = \frac{|A|}{|\Omega|}$

E.G. $A = \phi$: $\frac{|\phi|}{|\Omega|} = \frac{0}{2} = 0$

$A = \{H, T\}$: $\frac{|A|}{|\Omega|} = \frac{2}{2} = 1$

PRINCIPLE OF COUNTING

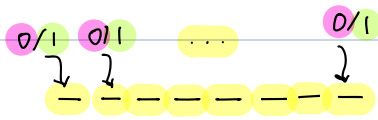
- TOSS A COIN $\Omega_1 = \{H, T\}$ $|\Omega_1| = 2$
- ROLL A DIE $\Omega_2 = \{1, 2, 3, 4, 5, 6\}$ $|\Omega_2| = 6$

TOGETHER : H1 H2 H3 H4 H5 H6

$\Omega = \Omega_1 \times \Omega_2$ T1 T2 T3 T4 T5 T6

$|\Omega| = |\Omega_1| \times |\Omega_2| = 2 \times 6 = 12$

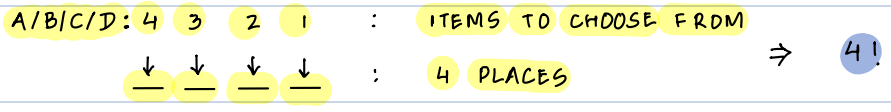
- FIND THE NUMBER OF POSSIBLE 8-BIT BYTES.



$2 \cdot 2 \cdots 2$

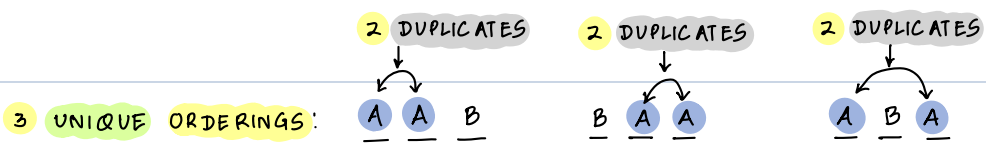
$= 2^8 = 256$

- FIND THE NUMBER OF FOUR LETTER SEQUENCES THAT CAN BE OBTAINED BY ORDERING THE LETTER A, B, C, D, WITHOUT REPETITION.



IN GENERAL, n ITEMS ⇒ $n!$ (PERMUTATIONS) WAYS TO ORDER WITHOUT REPETITIONS

- HOW MANY ORDERINGS OF LETTERS IN AAB ARE THERE, IF WE DO NOT DISTINGUISH BETWEEN THE As?



IF WE DISTINGUISH B/W As, # OF ORDERINGS: $3! = 6$ PERMUTATIONS : OVER-COUNT

TO OBTAIN THE UNIQUE (NON-DUPLICATE) ORDERINGS : DIVIDE THE # OF PERMUTATIONS BY THE WAYS YOU CAN SHUFFLE THE NON-DISTINCT / REPEATED LETTERS

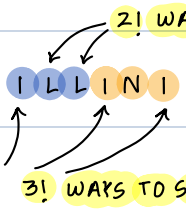
$2!$ WAYS TO SHUFFLE As IN EACH UNIQUE ORDERING ⇒ $\frac{3!}{2!}$: EXACT COUNT

PRINCIPLE OF OVER COUNTING

$\frac{3!}{2!}$ ← ADJUSTMENT FOR OVER COUNTING

- HOW MANY ORDERINGS OF LETTERS IN I L L I N I ARE THERE, IF WE DO NOT DISTINGUISH BETWEEN THE IS AND THE LS.

PRINCIPLE OF OVER COUNTING



: 6! PERMUTATIONS : OVER COUNT

$\Rightarrow \frac{6!}{3!\,2!}$: EXACT COUNT

- A, B, C, D, E, F, G, H, I : 9 BASKETBALL PLAYERS

HOW MANY LINEUPS (SUBSETS) OF 5 PLAYERS CAN BE THERE?

9 8 7 6 5 : # OF CHOICES
↓ ↓ ↓ ↓ ↓ : 5 SPOTS TO FILL

$\Rightarrow$ $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$ # OF LINEUPS INCLUDING DUPLICATES : OVER COUNT

E.G. A B C D E B A C D E ARE COUNTED TWICE BUT ARE THE SAME

IN TOTAL, ADJUST FOR 5! SHUFFLES!

OVER COUNT

$\Rightarrow \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{5!}$

COMING SOON! 9 CHOOSE 5

SAME AS $\binom{9}{5}$ DEFINED AS

$\binom{9}{5} := \frac{9!}{5!\,(9-5)!}$

$= \frac{9!}{5!\,4!} = \frac{9 \cdot 8 \cdot 7 \cdot 5 \cdot 4!}{5!\,4!}$

- TERMINOLOGY :

  - SEQUENCE : A COLLECTION OF OBJECTS IN AN ORDER
  - SET : A COLLECTION OF OBJECTS WHERE ORDER DOES NOT MATTER

E.G. (A B C) (A C B) (B A C) (B C A) (C A B) (C B A) : 6 (3!) DIFFERENT SEQUENCES

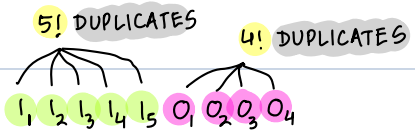
{A B C} {A C B} {B A C} {B C A} {C A B} {C B A} : ALL ARE THE SAME!

- HOW MANY BINARY SEQUENCES OF LENGTH 9 HAVE EXACTLY 5 ONES?

EXAMPLE SEQUENCE : 1 1 1 1 1 0 0 0 0

POSSIBLE SEQUENCES WITH DUPLICATES: # OF PERMUTATIONS OF 9 ITEMS (5 ONES AND 4 ZEROS) = $9!$

$9!$ OVER COUNT

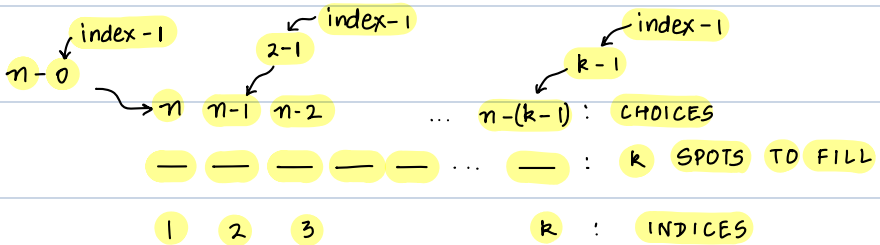


$\Rightarrow$ $\frac{9!}{5!\,4!}$: EXACT COUNT

WHAT WE DID IS SAME AS CHOOSING 5 ITEMS FROM SOME NINE ITEMS! $\binom{9}{5}$

IN GENERAL, IN HOW MANY WAYS ONE CAN CHOOSE k ITEMS FROM n?

n ITEMS → k ITEMS



$\Rightarrow$ $n(n-1) \dots n-(k-1)$: OVER COUNT

$\Rightarrow$ $n(n-1) \dots n-(k-1) \,/\, k!$: EXACT COUNT

$k!$ REMOVE DUPLICATES

$$= \frac{n(n-1)\dots n-(k-1)}{k!} \cdot \frac{(n-k)!}{(n-k)!}$$

$$= \frac{n!}{k!\,(n-k)!} \quad \text{DEFINITION} := \binom{n}{k} \text{ : } n \text{ choose } k \text{ : BINOMIAL COEFFICIENT}$$

- REMARK: $\binom{n}{k} = \binom{n}{n-k}$

- PROOF: LHS: $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

$$= \frac{n!}{(n-k)!\,(n-(n-k))!}$$

$$= \binom{n}{n-k} : \text{RHS}$$

HENCE THE REMARK.

→ PROBABILITY EXPERIMENTS WITH EQUALLY LIKELY OUTCOMES

- 9 SOCKS : 6 ORANGE 3 BLUE

SELECT 2 AT RANDOM!

WHAT IS THE PROB. THE TWO SOCKS ARE THE SAME COLOR?

TOTAL WAYS TO SELECT 2 OUT OF 9 : $\binom{9}{2} = \frac{9 \cdot 8}{2} = 36$

↑ EXHAUSTIVE # OF OUTCOMES

OF WAYS TO GET 2 ORANGES : $\binom{6}{2}$

OF WAYS TO GET 2 BLUES : $\binom{3}{2}$

⇒ FAVORABLE # OF OUTCOMES : $\binom{6}{2} + \binom{3}{2}$

$$\therefore \text{PROB} = \frac{\binom{6}{2} + \binom{3}{2}}{\binom{9}{2}}$$