ECE 313 / MATH 362 PROBABILITY WITH ENGINEERING APPLICATIONS

LECTURE 21: CONTINUOUS-TYPE RANDOM VARIABLES AND CUMULATIVE DISTRIBUTION FUNCTIONS

- TOPICS TO COVER (BASED ON CH 3.1)

→ CONTINUOUS-TYPE RANDOM VARIABLES AND CUMULATIVE DISTRIBUTION FUNCTIONS

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- CUMULATIVE DISTRIBUTION FUNCTIONS (CDFs):

DEF. FOR ANY RV X DEFINED OVER $(\Omega, \mathcal{F}, P)$, DEFINE CDF OF X AT c:

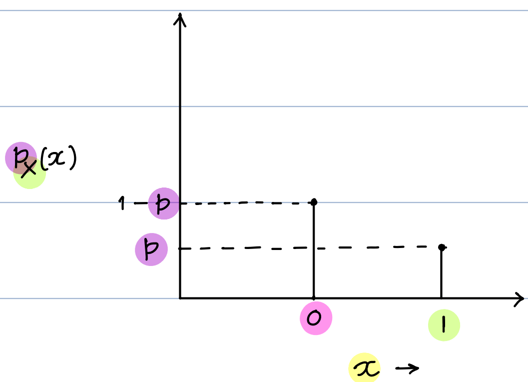
$$F_X(c) = P\{X \le c\}, \quad c \in \mathbb{R}$$

$$= P\{\omega : X(\omega) \le c\}$$

E.G. $X \sim \text{Bernoulli}(p)$ E.G. $p \le 1/2$

SUPPORT OF X OR $p_X(x)$

$$P\{X = x\} = p_X(x) = \begin{cases} p^x (1-p)^{1-x}, & x = 0, 1, \\ 0, & \text{otherwise.} \end{cases} \quad x \in \mathbb{R}$$



OBSERVATION: $\forall\ c_1 \le c_2$; $c_1, c_2 \in \mathbb{R}$

$F_X(c_1) \le F_X(c_2) \Rightarrow F_X$ IS NON-DECREASING

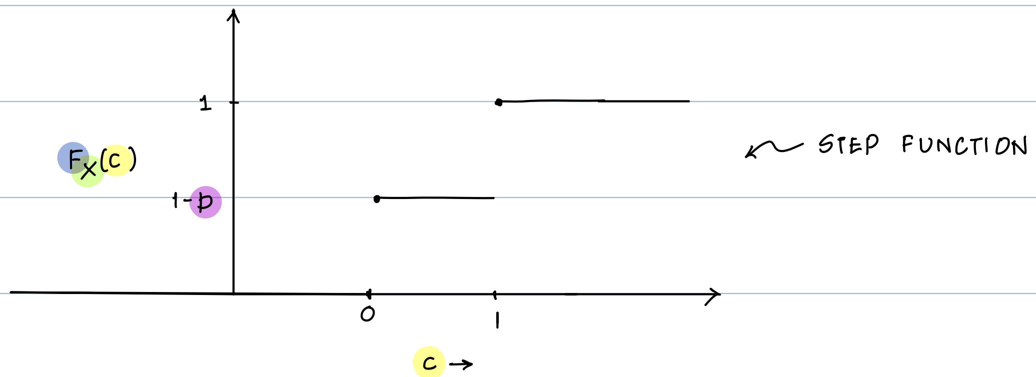
$F_X(c) := P\{X \le c\}$

TAKE $c = -1$: $F_X(-1) = P\{X \le -1\} = 0$

$c = 0$: $F_X(0) = P\{X \le 0\} = P\{X = 0\} = 1 - p$

$c = .5$: $F_X(.5) = P\{X \le .5\} = P\{X = 0\} = 1 - p$

$c = 1$: $F_X(1) = P\{X \le 1\} = P\{X = 0\} + P\{X = 1\} = 1 - p + p = 1$



FOR DISCRETE-TYPE RVs : $F_X(c) = P\{X \le c\} = \sum_{x \le c} p_X(x)$

$P\{X = x\}$

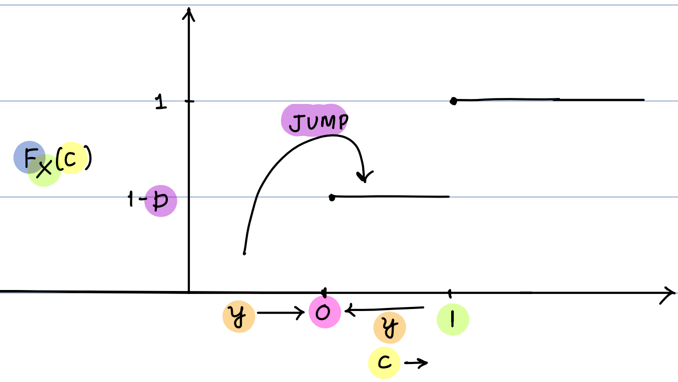
LEFT LIMIT AND RIGHT LIMIT OF A FUNCTION :

$F(\cdot)$ IS A REAL VALUED FUNCTION

$$F(x-) := \lim_{\substack{y \to x \\ y < x}} F(y) \qquad F(x+) := \lim_{\substack{y \to x \\ y > x}} F(y)$$

LEFT LIMIT OF F AT x

RIGHT LIMIT OF F AT x



$$F_X(0-) = \lim_{\substack{y \to 0 \\ y<0}} F(y) = 0 \neq F_X(0)$$

$$F(0+) := \lim_{\substack{y \to 0 \\ y>0}} F(y) = 1-p = F_X(0)$$

$$\Rightarrow \quad \underbrace{F_X(x)}_{\text{F AT } x} = \underbrace{F_X(x+)}_{\text{RIGHT LIMIT OF F AT } x} \neq \underbrace{F_X(x-)}_{\text{LEFT LIMIT OF F AT } x}$$

FOR BERNOULLI DIST.

- F IS NOT CONTINUOUS (AT x)
- F IS RIGHT CONTINUOUS (AT x)

SAME AS $F_X(x+)$

DEFINE $\underbrace{\Delta F_X(x)}_{\text{SIZE OF THE JUMP}} = F_X(x) - F_X(x-)$: JUMP AT x

$P\{X < c\}$

$p_X(x)$: CAN YOU SEE THIS?

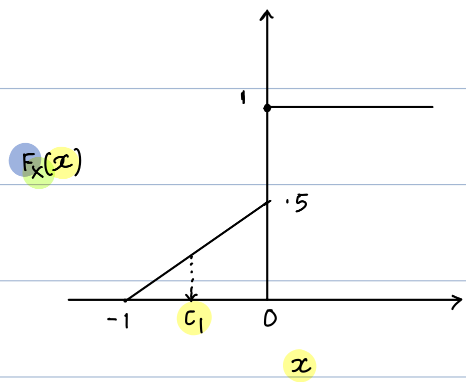
$P\{X \leq x\}$

WHY? DEFINITION!

OBSERVATIONS:

- $P\{X < c\} = F_X(x-)$
- $p_X(x) = P\{X = x\} = F_X(x) - F_X(x-)$: SIZE OF THE JUMP AT x
- $\lim_{c \to \infty} F_X(c) = 1 \qquad \lim_{c \to -\infty} F_X(c) = 0$

E.G.



- DETERMINE ALL u SUCH THAT $P\{X = u\} > 0$

$$p_X(0) = F_X(0) - F_X(0-) = \cdot 5$$

$c_1 \in (-\infty, 0)$: $p_X(c_1) = F_X(c_1) - F_X(c_1-) = 0 \qquad \because F$ IS CONT. IN $(-\infty, 0)$

ALL u SUCH THAT $P\{X = u\} > 0$: $u \in \{0\}$

- FIND $P\{X \leq 0\} = F_X(0) = 1$
- FIND $P\{X < 0\} = F_X(0-) = \cdot 5$

PROPOSITION. A FUNCTION F IS THE CDF OF SOME RV IF AND ONLY IF :

F.1 F IS NONDECREASING.

F.2 $\lim_{c \to +\infty} F(c) = 1$, $\lim_{c \to -\infty} F(c) = 0$.

F.3 F IS RIGHT CONT., i.e., $F_X(c) = F_X(c+) \quad \forall c \in \mathbb{R}$.