University of Illinois **Fall 2025**

ECE 313: Problem Set 8

Due: Friday, Oct 31 at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.5 - 3.6.2

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Mondays, and is due by 11:59 p.m. on the following Monday. You must upload handwritten homework to Gradescope. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON Gradescope
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. [**Customers in a coffee shop**]
   Alice is working in a coffee shop. She found that Customer arrivals follow a Poisson process with rate $\lambda = 4$ customers/hour.
   (a) What is the probability that no customer arrives within the next 30 minutes?
   (b) Alice wants to take a 30-minute nap during work. She will be reported if any customer arrives during the nap. When should Alice start her 30 mins nap to minimize the chance of being caught? Assume there's no customer at the shop right now at $T = 0$.
   (c) If Alice wants the probability of being reported to be less than 50%, how long can she sleep?

2. [**Poisson Process Intervals**]
   Let N_t be a Poisson process with rate $\lambda > 0$. Your answers may include λ for the following questions.
   (a) Find $P(N_5 = 7)$.
   (b) Find $P(N_8 - N_3 = 7)$ and $E[N_8 - N_3]$.
   (c) Find $P(N_8 - N_3 = 7 | N_5 - N_4 = 5)$.
   (d) Find $P(N_5 - N_4 = 5 | N_8 - N_3 = 7)$.

3. [**Communication in Gaussian Noise**]
   A wireless communication system consists of a transmitter and a receiver. The transmitter sends a signal x, and the receiver observes

$$Y = x + Z,$$

   where Z is a noise term, modeled as a Gaussian random variable with mean $\mu_Z = 0$ and variance $\sigma_Z^2 = 2$.

(a) Suppose the transmitted signal is $x = 1$. What is the pdf of the received signal Y?

(b) Now suppose the transmitted signal can be either $x = -1$ or $x = 1$. The receiver uses the following decoding rule: if $Y > 0$, it declares that $x = 1$; if $Y \leq 0$, it declares that $x = -1$. Assuming that the transmitter sends -1 or $+1$ with probability $1/2$ each, what is the receiver's error probability in terms of Q? (P.S. This modulation is called BPSK)

(c) Now suppose the transmitted signal x can be chosen from three possible values: $x = -2$, $x = 0$ and $x = 2$. The receiver now uses the following decoding rule: if $Y < -1$, it declares $x = -2$, if $Y > 1$, it declares $x = 2$, and otherwise it declares $x = 0$. Assuming the transmitter sends each possible symbol with probability $1/3$, what is the receiver's error probability?

4. [**Scaling of distributions**]

(a) Assume that you have a random number generator that produces a uniformly distributed random variable X over the interval $[-2, 6]$. Find a linear function g such that $g(X) = Y$, where Y is uniformly distributed over the interval $[2, 10]$.

(b) Solve part (a) with $X \sim \mathcal{N}(-2, 4)$ and $Y \sim \mathcal{N}(1, 2)$.