University of Illinois **Fall 2025**

ECE 313: Problem Set 7

Due: Friday, October 24 at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.4 - 3.6.1

Note on reading: For most sections of the course notes, there are short-answer questions at the end of the chapter. We recommend that after reading each section, you try answering the short-answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted. Please write at the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned pages.**

1. **[Using a CDF]**
   Let X be a random variable with a CDF defined as follows:

$$F_X(c) = \begin{cases} 0 & c < -10 \\ \frac{1}{2} & -10 \le c < -5 \\ \frac{2}{3} & -5 \le c < 0 \\ \frac{4}{5} & 0 \le c < 5 \\ 1 & c \ge 5 \end{cases}$$

   Sketch the CDF and compute the following values:

   (a) $P(X \le 3.5)$
   (b) $P(X \ge -4.37)$
   (c) $P(|X| < 2)$
   (d) $P(X^2 \le 9)$
   (e) $\mathbb{E}[X]$
   (f) $\mathrm{Var}(X)$

2. **[Exponential Distribution]**
   Suppose X has an exponential distribution with parameter $\lambda > 0$. Answer the following in terms of λ.

   (a) Determine $E[X^2]$.
   (b) Determine $P\{\lfloor X^2 \rfloor = 4\}$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

3. **[Random Variable Summation]**
   Let $X \sim \text{Uniform}(0,1)$ uniformly distributed over the unit interval. $Y \sim \text{Binomial}(n=2, p=0.4)$ is a discrete random variable following the binomial distribution. Assume X and Y are independent. Define a new random variable $Z = X + Y$.

   (a) Find $F_Y(k)$ (CDF of Y)

   (b) Compute $P\{Z \leq 1.5\}$

   (c) Find $F_Z(k)$ (CDF of Z)

   (d) Find $f_Z(k)$ (PDF of Z)

4. **[Poisson Process]**
   Let $N = (N_t;\, t \geq 0)$ be a Poisson process with rate $\lambda = 2$. Find:

   (a) $P(N_1 \geq 1 \mid N_2 = 2)$

   (b) $P(N_2 = 2 \mid N_1 \geq 1)$

5. **[Scaling PDFs]**
   Suppose that X and Y are the sampled values of two different audio signals. The mean and variance of an audio signal are uninteresting: the mean tells you the bias voltage of the microphone, and the variance tells you the signal's loudness. For this reason, the audio signals X and Y are pre-normalized so that $E[X] = E[Y] = 0$ and $\text{Var}(X) = \text{Var}(Y) = 1$.

   An audio signal Z is said to be "spiky" if $P\{|Z| > 3\sigma_Z\} > 0.01$, i.e., one-in-hundred samples has a large amplitude.

   Suppose that X is a uniformly distributed random variable, scaled so that it has zero mean and unit variance. (1) What is $P\{|X| > \sigma_X\}$? (2) What is $P\{|X| > 3\sigma_X\}$? (3) Is X spiky? Be sure to consider both positive and negative values of X.