ECE 313: Problem Set 9

Due: *Monday* October 31 at 11:59 pm
Reading: *ECE 313 Course Notes*, Sections 4.1-4.3

1. **[Joint PMF]**
   An IC manufacturing company is testing six IC chips they have created, two of which are defective. To determine which chips are defective, they test each of them individually one at a time. Let M_1 be the number of chips tested until the first defective is identified. Let M_2 be the number of additional chips tested until the second defective chip is detected.

   (a) Find the joint pmf, $p_{M_1,M_2}(i,j)$, for all i and j.
   (b) Find the marginal pmf $p_{M_1}(i)$.
   (c) Find the marginal pmf $p_{M_2}(j)$.
   (d) Are M_1 and M_2 independent? Justify your answer.
   (e) Find $P(M_2 > M_1)$.
   (f) Find $P(M_2 = M_1)$.
   (g) Find $P(M_1 = 4|M_2 = 1)$.

2. **[Joint CDF]**
   Consider two random variables X,Y with the joint CDF given by.

   $$F_{X,Y}(x,y) = \begin{cases} 1\text{-e}^{-x} - e^{-y} + e^{-x-y} & x,y \geq 0 \\ 0 & \text{else} \end{cases}$$

   (a) Find $P(1 < X < 3, 1 < Y < 3)$.
   (b) Find $P(X > 2 \text{ or } Y > 2)$.
   (c) Find $P(X < 1, Y < 1|X < 3, Y < 3)$

3. **[Joint probability distribution]**
   The joint pdf of random variables X, Y is given by

   $$f_{X,Y}(x,y) = \begin{cases} cxy & x,y \geq 0,\ 0 \leq x+y \leq 2 \\ 0 & \text{else} \end{cases}$$

   (a) Find c.
   (b) Find the marginal pdf $f_X(x)$.
   (c) Find $E[X]$ and $\text{Var}(X)$.
   (d) Find the marginal pdf $f_Y(y)$.
   (e) Find $E[Y]$ and $\text{Var}(Y)$.

4. **[Joint probability distribution]**
   The joint pdf of X, Y is given by

   $$f_{X,Y}(x,y) = \begin{cases} c(x+y) & 0 \leq x,y \leq 1 \\ 0 & \text{else} \end{cases}$$

   (a) Find c.
   (b) Let $Z = \min\{X, 2Y\}$. Find $f_Z(z)$, the pdf of Z.