

ECE 313: Problem Set 8

Due: *Monday* October 24 at 11:59 pm

Reading: *ECE 313 Course Notes*, Sections 3.8-3.10

1. **[Gaussian Approximation]**

Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$ using the normal approximation for a binomial distribution with continuity correction. Please use the Normal tables in the course notes to simplify your final solution.

2. **[Distribution of a function of uniformly distributed random variable]**

Let X be a random variable uniformly distributed in $[0, 1]$ and let Θ be a random variable uniformly distributed in $[-\pi/2, \pi/2]$.

- (a) Find the CDF, pdf and expected value of the random variable $Y = X^n$.
- (b) Find the CDF and pdf of the random variable $Y = \tan(\Theta)$. Note that the tan function increases from $-\infty$ to ∞ when the angle increases from $-\pi/2$ to $\pi/2$.
- (c) Find the mean of $Y = \cos(\Theta)$.

3. **[Distribution of a function of an exponential random variable.]**

X is an exponential random variable with mean 1. Let $Y = (1 + X)^2$.

- (a) Find the CDF of Y .
- (b) Find the pdf of Y .
- (c) Calculate $E[Y]$.

4. **[Absolute value of a random variable]**

Express the pdf of $|Y|$ in terms of the pdf of Y .

5. **[Hypothesis Testing]**

In a communication system, a bit $X \in \{0, 1\}$ is transmitted. The received signal $Y = X + W$ where the random noise W is standard normal (i.e., Gaussian distribution with mean 0 and variance 1). The goal is to make a decision on whether X is 0 or 1 based on the received signal Y .

- (a) Suppose X is equally likely to be 0 or 1. Find the MAP decision rule to decide if X is 0 or 1.
- (b) Now suppose that $P[X = 0] = 0.3$ and $P[X = 1] = 0.7$. Find the MAP decision rule to decide if X is 0 or 1.