ECE 313: Problem Set 7

Due: *Monday* October 17 at 11:59 p.m.
Reading: *ECE 313 Course Notes*, Sections 3.6 and 3.7

1. [**Gaming**]
The amount of time it takes to finish a round of an online strategy game is an exponentially distributed random variable with parameter $\lambda > 0$. Suppose that a user has played 10 rounds over the span of 5 hours. Assuming the time it takes to finish each round is independent of each other, find the maximum likelihood estimator of λ.

2. [**iflag Phone Company**]
The iflag phone company recently released their flagship phone *iflag 14*. The company offers a full refund for any phone malfunctioning in its first year of operation. According to the historical data, the company expects to sell 10^6 phones during 2022-2023. Suppose that each phone malfunctions during the first year of its operation with probability $p \in (0, 1)$, independent of anything else.
   (a) Given that $p = 2 \times 10^{-6}$, what is the probability that the company will issue more than 5 refunds? (Leave your answer as a summation)
   (b) Use the Markov inequality to provide a bound for part (a).
   (c) Use Chebyshev's inequality to provide an upper bound for part (a).
   (d) Use Poisson approximation to solve (a). (Report your number)
   (e) Use Gaussian approximation with continuity correction to solve (a). (Report your number using Normal tables on Page 245-246 of the Course book, Hint: since p is very small, you can approximate $1 - p$ with one.)
   (f) Suppose that at the end of the year, the company issued 20 refunds. Using Gaussian approximation with continuity correction, estimate $\widehat{p}_{\mathrm{ML}}$. (Hint: since p is very small, you can approximate $1 - p$ with 1 in your derivation. Root of $\frac{\mu+20.5}{\mu+19.5} - \exp\left((20-\mu)/\mu\right)$ is 19.5062.)

3. [**A Relatively Old Country**]
The age of a randomly selected individual in the USA is a Gaussian random variable with mean $\mu = 39.1$ years and standard deviation $\sigma = 5$ years. (Report your numbers using Normal tables on Page 245-246 of the Course book)
   (a) Find the probability that a randomly selected individual is at least 50.
   (b) Find the probability that a randomly selected individual is at least 30, and at most 50 years old.
   (c) Find the age $k > 0$ so that the 90% of the population is at least k years old (k can be any positive number).