University of Illinois **Fall 2022**

ECE 313: Problem Set 5

Due: *Tuesday* September 27 at 11:59 p.m.
Reading: *ECE 313 Course Notes*, Sections 2.11

1. **[House phone number]**
   You move into a new house; the phone is connected. You are sure that the first 6 digits are 217-383, but not certain about the last 4 digits. You think the phone number might be 217-383-3428. As an experiment, you pick up the phone and dial 217-383-3428 at 6am. You obtain a busy signal. Note that the phone line will be busy if you call the same phone. For invalid numbers, you get an error message instead of busy line signal. Suppose that the total number of valid phone numbers that start from 217-383 in Champaign IL is 500. Are you now more sure of your phone number? If so, how much? What assumptions do you make to reach the conclusion?

2. **[Matching Bernoulli parameters]**
   Consider hypotheses H_0 and H_1 about a two dimensional observation vector $X = (X_1, X_2)$. Under H_0, X_1 and X_2 are independent and identically distributed. Both have the Bernoulli distribution with $p = 0.5$. Under H_1, X_1 and X_2 are mutually independent, X_1 has the Bernoulli distribution with mean $p = 0.2$, and X_2 has the Bernoulli distribution with mean $p = 0.8$.

   (a) Describe the maximum likelihood rule for deciding which hypothesis is true.

   (b) Describe the MAP rule for deciding which hypothesis is true, assuming the prior distribution with $\frac{\pi_0}{\pi_1} = \frac{1}{2}$.

3. **[A bent coin]**
   Suppose you keep flipping a coin until you observe 3 heads. The random variable X is the number of flips that is required. Based on the observation, you need to choose one of the following two hypothesis: H_0: it is a fair coin with $P(H) = 0.5$, and H_1: the coin is bent with $P(H) = \frac{2}{3}$.

   (a) Describe the ML decision rule. Express it in a simplified form. (Hint: $\frac{\log 8}{\log 1.5} = 5.13$.)

   (b) Describe the MAP decision rule under the assumption that H_0 is a priori twice as likely as H_1. Express it in a simplified form. (Hint: $\frac{\log 16}{\log 1.5} = 6.84$.)

   (c) Find the average error probability, p_e, for the ML rule, using the same prior distribution given in part (b)

   (d) Find the average error probability, p_e, for the MAP rule, using the same prior distribution given in part (b).

4. **[True or false questions]**
   Consider a binary hypothesis testing problem with H_0: X follows a geometric distribution with parameter $p = 0.5$, and H_1: X follows a geometric distribution with parameter $p = 0.2$. Please state whether the following statements are true or false and provide reasoning.

   (a) If the priors $\pi_0 = \pi_1$, then the ML and the MAP estimators are the same.

   (b) If the ML decision rule is employed, then $p_{\text{false alarm}} > p_{\text{miss}}$.

   (c) MAP decision rule always provides lower $p_{\text{false alarm}}$ than ML decision rule.