Lecture 2 - 08/24

Review:

① uncertainty is everywhere

② Set operations. $A=\{1,2,3\}$, $B=\{3,4\}$, $\Omega=\{1,2,3,4,5\}$

Main set operations: $A\cup B=\{1,2,3,4\}$. $A\cap B=\{3\}$. $A^c=\{4,5\}$

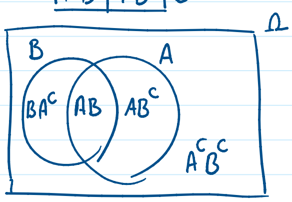
Other notations: $A-B=A\setminus B=A\cap B^c=\{1,2\}$, $|A|=3$, $\emptyset$, 2^Ω

Demorgan's law: $(A\cup B)^c=A^c\cap B^c$, $(A\cap B)^c=A^c\cup B^c$

Karnaugh map:

	A^c	A
B	A^cB	AB
B^c	A^cB^c	AB^c

Venn diagram:



③ Probability space.

- experiment. (Ω, P, F)
  - Ω set of possible outcomes
  - F set of observables of experiment which are subsets of Ω
  - P probability measure which is a function from F to $[0,1]$

④ Event axioms:

E.1.: $\Omega\in F$

E.2.: $A\in F \Rightarrow A^c\in F$

E.3. $A,B\in F \Rightarrow A\cup B\in F$. More generally

$$A_1, A_2, \ldots \in F \Rightarrow A_1\cup A_2\cup\cdots \in F$$

An example:

Experiment: quality-check division recieve 2 IC's and test them.

Results reveals: {IC is OK} OR {IC Overheats} OR {IC does not work} and reports to manager whether IC is usable or not.

. From quality check point of view:

$$\Omega = \{OK, Heat, Not\ work\}$$

$$\mathcal{F} = 2^{\Omega} = \text{set of all possible outcomes} = 8 \text{ members}$$

. From manager point of view:

$$\Omega = \{OK, Heat, Not\ work\}$$

$$\mathcal{F} = \{OK, Not\ OK, \Omega, \emptyset\}$$

Today:

① Probability axioms and implications

② Calculating size of sets

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① Probability axioms.

P.1: $A \in \mathcal{F}$, $P(A) \geq 0$

P.2: $A, B \in \mathcal{F}$ and $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$

More generally if $A_1, A_2, \ldots \in \mathcal{F}$ then $P(A_1 \cup A_2 \cup \ldots) = \sum_{i=1}^{\infty} P(A_i)$

P.3. $P(\Omega) = 1$

Implications:

P.4. $P(A \cup A^c) = P(A) + P(A^c) \Rightarrow P(A^c) = 1 - P(A)$

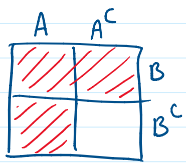
P.5. $P(A) \in [0,1] = \{x : 0 \leq x \leq 1\}$

P.6. $P(\emptyset) = 0 = P(\Omega^c)$

P.7. $A \subset B \Rightarrow P(B) = P(A \cup B) = P(A) + P(A^c B) \Rightarrow P(A) \leq P(B)$

P.8. $P(A \cup B) = P(A^c B) + P(AB) + P(AB^c)$

$= P(B) + P(A) - P(AB)$



P.9. Ex. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$

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② calculating size of sets.

If each element of Ω have equal probability (Ω finite) and $\mathcal{F} = 2^{\Omega}$

$$P(A) = \frac{|A|}{|\Omega|} \quad \text{(by axiom P.2.)}$$

If each element of Ω have equal probability (Ω finite) and $P = 1$

$$P(A) = \frac{|A|}{|\Omega|} \quad \text{(by axiom P.2.)}$$

How to count # of elements in A?

Principles of counting:

If there are n way to select shirts and m ways to select trousers then there are mn ways to select shirt and trousers

Principle of sum:

If there are n ways to dress warm and m ways to dress cold there are $m+n$ ways to dress warm or cold.

e.1. How many numbers are there with 4 digits such that.

(a) no furthur constraint: always start with the constraint:

$$\underset{\substack{\text{1st digit}\\\text{cannot be 0.}}}{\underline{9}} \times \underset{\text{2nd digit}}{\underline{10}} \times \underset{\text{3rd}}{\underline{10}} \times \underset{\text{4th}}{\underline{10}}$$

, digits are numbered from left.

(b) the number is odd and repetition is not allowed:

$$\underset{\substack{\text{4th digit}\\\text{belongs to}\\\{1,3,5,7,9\}}}{\underline{5}} \times \underset{\substack{\text{1st digit}\\\text{Cannot be 0}\\\text{or equal to}\\\text{4th digit}}}{\underline{8}} \times \underset{\text{2nd digit}}{\underline{7}} \times \underset{\text{3rd digit}}{\underline{6}}$$

(c) number is even & repetition is not allowed.

Notice that if the 4th digit is 0, the 1st digit can be anything in $\{1,2,\dots,9\}$

On the other hand if the 4th digit is in $\{2,4,6,8\}$ 1st digit has 8 choices $\Rightarrow$ principle of sum.

① 4th digit is 0:

$$\underset{\text{4th digit}}{\underline{1}} \times \underset{\text{1st digit}}{\underline{9}} \times \underset{\text{2nd digit}}{\underline{8}} \times \underset{\text{3rd digit}}{\underline{7}}$$

② 4th digit is not 0:

$$\underline{4} \times \underline{8} \times \underline{8} \times \underline{7}$$

② 4th digit is not 0:

$$\underset{\text{4th digit}}{\underline{4}} \times \underset{\text{1st digit}}{\underline{8}} \times \underset{\text{2nd digit}}{\underline{8}} \times \underset{\text{3rd digit}}{\underline{7}}$$

$\Rightarrow$ # number $= 9 \times 8 \times 7 + 4 \times 8 \times 8 \times 7$

Selecting members from a set:

$\binom{n}{k}$ = # of way to select k elements from a set of size n $= \dfrac{n!}{k!(n-k)!} = \dfrac{n \times (n-1) \times \cdots \times (n-k+1)}{k!}$

Principle of overcounting:

if each element of a set is counted L times, # of elements in set equal $\dfrac{\text{total count}}{k!}$