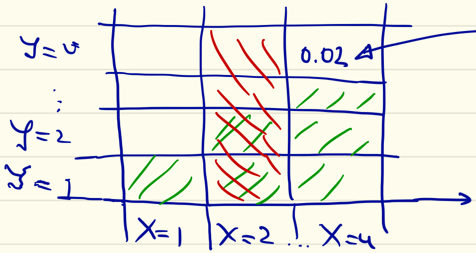
ECE 313: Lecture 27
Joint CDFs (Ch 4.1)
Joint pmfs (Ch 4.2)

Recall: joint distribution (pmf) of X & Y r.v. of discrete-type



$$P\{X = u, Y = v\} = p_{X,Y}(u, v)$$

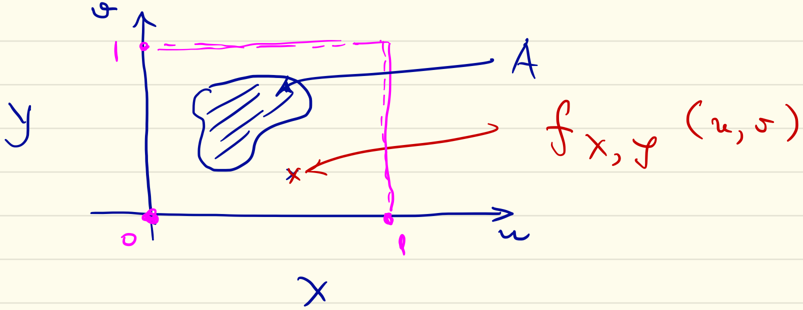
$$P\{\underbrace{(X, Y)}\in A\} = \sum\sum_{(u,v)\in A} p_{X,Y}(u, v)$$

Ex$_1$: $P(X \overset{A}{\geq} Y)$

Ex$_2$: $P\{X = 2\} = \sum_{v} p_{X,Y}(u, v) = p_X(2)$

marginal prob

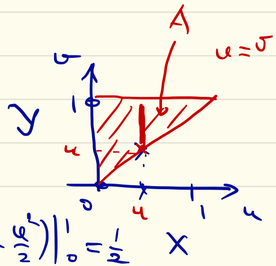
Joint pdf : X & Y are continuous-type r.v.



Ex 3.a $\quad f_{X,Y}(u,v) = \begin{cases} 1 & \text{if } 0 \le u \le 1,\ 0 \le v \le 1 \\ 0 & \text{else} \end{cases}$

$$P\{(X,Y) \in A\} = \iint\limits_{A} f_{X,Y}(u,v)\, du\, dv$$

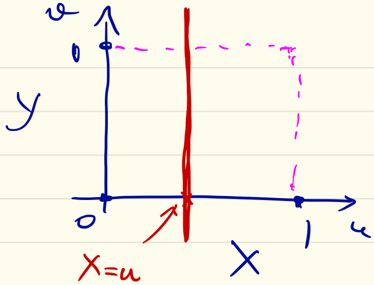
Ex 3.b. $\quad P\{\underbrace{X \le Y}_{\in A}\} = \iint\limits_{A} 1 .\, du\, dv = \int_0^1 du \left(\int_u^1 dv \right)$



$$= \int_0^1 du \, \left[v \right]_u^1 = \int_0^1 (1-u)\, du = \left(u - \frac{u^2}{2} \right) \Big|_0^1 = \frac{1}{2}$$

(*) Def
Marginal distribution

$$f_X(u) \overset{def}{=} \int_{-\infty}^{+\infty} f_{X,Y}(u,v)\, dv$$



Ex: (3e) :

$$f_X(u) = \int_{-\infty}^{+\infty} f_{X,Y}(u,v)\, dv$$

for $0 \leq u \leq 1 \longrightarrow$ $$= \int_0^1 1 \cdot dv = 1$$

else $$= 0$$

(*) Def : Conditional distribution

$$f_{Y|X}(v \mid X=u) = f_{Y|X}(v \mid u) = \frac{f_{X,Y}(u,v)}{f_X(u)}$$

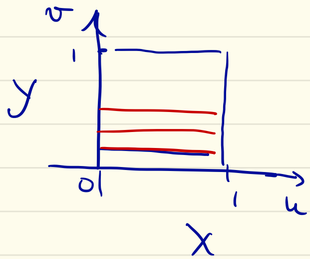
(Similar to $P(B \mid A) = \dfrac{P(A,B)}{P(A)}$)

Ex 4:

$$f_{X,Y}(u,v) = \begin{cases} c\,(1+u+v+uv) & 0 \le u,v \le 1 \\ 0 & \text{else} \end{cases}$$

Condition on c:

$$\underbrace{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{X,Y}(u,v)\, du\, dv}_{\parallel} = 1$$



$$= \int_0^1 \int_0^1 c\,(1+u+v+uv)\, du\, dv$$

$$= c \cdot \Big(\underbrace{\int_0^1\int_0^1 1\, du\, dv}_{} + \int_0^1\int_0^1 u\, du\, dv + \iint v + \iint uv \Big)$$

$$\left(\int_0^1 1\, du\right)\left(\int_0^1 1\, dv\right) + \left(\int_0^1 u\, du\right)\left(\int_0^1 1\, dv\right) \qquad \left(\int_0^1 u\, du\right)\left(\int_0^1 v\, dv\right)$$

separable functions & region

$$= c\left(1 \cdot 1 + \frac{1}{2} \cdot 1 + 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}\right)$$