**University of Illinois** Fall 2017

# ECE 313: Hour Exam I

Wednesday, October 11, 2017
8:45 p.m. — 10:00 p.m.

**Name: (in BLOCK CAPITALS)** ______________________________

**NetID:** ____________ **Signature:** ______________________

**Section:** ☐ A, 9:00 a.m. ☐ B, 10:00 a.m. ☐ C, 11:00 a.m. ☐ D, 1:00 p.m. ☐ E, 2:00 p.m.

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, cellphones, headphones, etc. are not allowed.

The exam consists of **nine** problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page. Draw a small box around each of your final numerical answers.

Grading

1. 6 points ____________
2. 10 points ____________
3. 6 points ____________
4. 12 points ____________
5. 12 points ____________
6. 16 points ____________
7. 10 points ____________
8. 16 points ____________
9. 12 points ____________

Total (100 points) ____________

1. **[6 points]** Suppose that a fair coin is independently tossed twice. Define the events:

$$A = \{\text{“The first toss is a head”}\}$$
$$B = \{\text{“The second toss is a head”}\}$$
$$C = \{\text{“Both tosses yield the same outcome”}\}.$$

Are $A, B, C$ independent? Please show your work.

2. **[10 points]** Suppose a coin is bent or fair with equal probability. In particular, let the probability of showing up head be $p$, then $P(p = 0.5) = P(p = 0.8) = 0.5$. We toss the coin twice.

   (a) Find the probability of getting two heads.

   (b) We will decide if the coin is fair based on the number of heads out of two tosses. How many different decision rules are there? (Hint: The decision rule does not need to be MAP or ML)

3. **[6 points]** A machine produces computer chips that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced chips get passed through a quality control equipment, which is able to detect any chip that is obviously defective and discard it. What is the probability that a chip is good given that it made it through the quality control equipment?

4. **[12 points]** Consider a computational creativity system like MasterProbo that can generate an infinite sequence of questions. A student uses the system and answers questions correctly with probability 4/5, independently of all other questions.

   (a) What is the probability the student answers exactly two of the next five questions incorrectly?

   (b) What is the probability that the number of questions the student needs to get three questions correct is five?

5. **[12 points]** *Ziziphus mauritiana*, also known as Chinese date, ber, Chinee apple, jujube, Indian plum, Regi pandu, and Indian jujube is a tropical fruit tree species belonging to the family *Rhamnaceae*. Some of its fruits are sweet and some are bitter (which we assume to be independent of one another); it is very difficult to tell without tasting. The probability of sweet is $p$ and the probability of bitter is $1 - p$. A forest dweller eats a very large sequence of fruits.

   (a) What is the probability the time until the first sweet fruit is six?

   (b) What is the expected number of sweet ones the forest dweller will get before he/she has gotten three bitter ones?

6. **[16 points]** Consider two hypotheses, $H_1$ and $H_0$. If $H_1$ is true, the observation $X$ has the distribution $P(X = 1) = 0.2$ and $P(X = 2) = 0.8$. If $H_0$ is true, $P(X = 1) = P(X = 2) = 0.5$.

   (a) Specify the ML decision rule given the observation $X$. What is the miss probability?

   (b) Given the prior distribution where $H_1$ is true with probability $\pi_1 = 0.8$. Specify the MAP decision rule given the observation $X$. What is the miss probability?

   (c) Given the same prior distribution as in part (b), suppose we have two independent observation of $X$, what is the new MAP decision rule based on two observations?

7. [**10 points**] Three prisoners A, B and C are sentenced to death. The governor, however, has selected one of them to be pardoned, each with equal probability. The warden knows who is to be pardoned, but is not allowed to tell. Instead the warden agrees to tell A one name that is not to be pardoned as follows.

   - If B is to be pardoned, the warden gives C's name.
   - If C is to be pardoned, the warden gives B's name.
   - If A is to be pardoned, the warden gives B or C's name with equal probability.

   Find the conditional probability that A is to be pardoned given the warden gives B's name.

8. **[16 points]** In basketball games players make three-point field goal attempts and some of the attempts result in successful three-point field goals. Assume each of the attempts of a player is successful independently with equal probability called the three-pointer shooting percentage. Different players may have different three-pointer shooting percentages.

(a) Alice has a three-pointer shooting percentage of 0.4 and successfully made one three-point field goal in a game. The number of three-point field goal attempts by Alice in this game is known to be less than or equal to three, but otherwise unknown. Give a maximum likelihood estimate of the number of her three-point field goal attempts.

(b) Let Bob's three-pointer shooting percentage $p$ be unknown. For practice Bob attempts $n$ three-point field goals and successfully makes $X$ of them. Let his three-pointer shooting percentage be estimated by $\hat{p} = \frac{X}{n}$. If $p$ is to be estimated within 0.05 (i.e., by an interval estimate of length 0.1) with 75% confidence, find the minimum value of $n$ based on

$$P\left\{p \in \left(\hat{p} - \frac{a}{2\sqrt{n}}, \hat{p} + \frac{a}{2\sqrt{n}}\right)\right\} \geq 1 - \frac{1}{a^2},$$

where $a$ can be any positive number.

9. **[12 points]** Suppose you want to divide a 52 card deck into four hands with 13 cards each. What is the probability that each hand has a king? (Hint: All 52 cards are distinct and there are only four kings in the set of the cards)