ECE 313: Final Exam

Monday, December 12, 2016
7 p.m. — 10 p.m.
Aa-Fh in room ECEB 1013
Fi-Zz in room ECEB 1002

1. [**14 points**] A drawer contains 4 black, 6 red, and 8 yellow socks. Two socks are selected at random from the drawer.

   (a) What is the probability the two socks are of the same color?
   Solution: Let B, R, and Y denote the sets of black, red, and yellow socks, with cardinalities 4, 6, and 8, respectively. A suitable choice of sample space for this experiment is

   $$\Omega = \{S : |S| = 2 \text{ and } S \subset B \cup R \cup Y\},$$

   where S represents the set of two socks selected. The cardinality of Ω is

   $$|\Omega| = \binom{4+6+8}{2} = \binom{18}{2} = 153.$$

   Let F be the event $F = \{S : |S| = 2 \text{ and } S \subset B \text{ or } S \subset R \text{ or } S \subset Y\}$. Then,

   $$|F| = \binom{4}{2} + \binom{6}{2} + \binom{8}{2} = 6 + 15 + 28 = 49.$$

   Thus,

   $$P(F) = \frac{49}{153}.$$

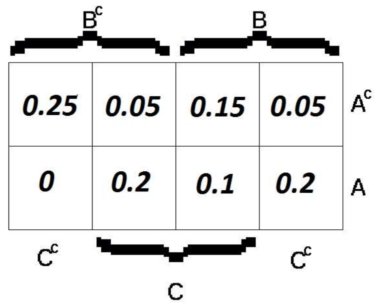
   (b) What is the conditional probability both socks are yellow given they are of the same color?
   Solution: Let $G = \{S : |S| = 2 \text{ and } S \subset Y\}$. Note that $G \subset F$ and $|G| = \binom{8}{2} = 28$. Therefore,

   $$P(G|F) = \frac{P(FG)}{P(F)} = \frac{P(G)}{P(F)} = \frac{28}{49} = \frac{4}{7}.$$

2. [**14 points**] The two parts of this problem are unrelated.

   (a) Suppose A, B, and C are events for a probability experiment such that B and C are mutually independent, $P(A) = P(B^c) = P(C) = 0.5$, $P(AB) = P(AC) = 0.3$, and $P(ABC) = 0.1$. Fill in the probabilities of all events in a Karnaugh map. Show your work AND use the map on the right to depict your final answer.

   Solution: Start by filling in $P(ABC) = 0.1$. Then use $P(AC) = 0.3$ to get $P(AB^cC) = P(AC) - P(ABC) = 0.3 - 0.1 = 0.2$. Similarly, use $P(AB) = 0.3$ to get $P(ABC^c) = P(AB) - P(ABC) = 0.3 - 0.1 = 0.2$. The independence of B and C and the given probabilities of B and C yield $P(BC) = P(B)P(C) = 0.25$, from which we conclude as before that $P(A^cBC) = P(BC) - P(ABC) = 0.25 - 0.1 = 0.15$. Use $P(A) = 0.5$ to get $P(AB^cC^c) = 0$; use $P(B) = 0.5$ to get $P(A^cBC^c) = 0.05$; use $P(C) = 0.5$ to get $PA^cB^cC) = 0.05$. Finally, all probabilities add to one;



$P(A^cB^cC^c) = 0.25$.

(b) Let A, B be two disjoint events on a sample space Ω. Find a formula for the probability of A occurring before B in an infinite sequence of independent trials.

Solution: A and B occur with probabilities $P(A)$ and $P(B)$, respectively. Consider the first trial:

First Trial: Either A occurs or B occurs or neither A nor B occurs.

- If A occurs, then the probability that A occurs before B is 1.
- If B occurs, then the probability that A occurs before B is 0.
- If neither A nor B occurs, then the process starts over.

Let s be the probability that neither A nor B occurs in a given independent trial. Then $s = 1 - P(A) - P(B)$ due to $A \cap B = \emptyset$. Therefore,

$$\begin{aligned} P(A \text{ before } B) &= P(A) + sP(A) + s^2P(A) + \cdots + s^nP(A) + \cdots \\ &= P(A)\sum_{n=0}^{\infty} s^n = P(A)\frac{1}{1-s} = \frac{P(A)}{P(A)+P(B)}. \end{aligned}$$

Alternatively: Let s be the probability that neither A nor B occurs in a given independent trial. If neither A nor B occurs on the first trial, then the process starts over. So $P(A \text{ before } B) = P(A) + sP(A \text{ before } B)$. Solving this equation for $P(A \text{ before } B)$ yields $P(A \text{ before } B) = \frac{P(A)}{1-s} = \frac{P(A)}{P(A)+P(B)}$.

3. **[20 points]** Suppose two teams, Cubs and Indians, play a best-of-seven series of games. Assume that games are independent, that ties are not possible in each game, and that Cubs wins a given game with probability $p \in (0, 1)$. The series ends as soon as one of the teams has won four games. Let G denote the total number of games played.

(a) Obtain the probability that Cubs win exactly 2 of the first 4 games.

Solution: The number of games that Cubs win out of the first 4 games is $Binomial(4, p)$, hence

$$P\{\text{Cubs win exactly 2 of the first 4 games}\} = \binom{4}{2}p^2(1-p)^2$$

(b) What is the expected number of games that Cubs will win out of the first 4 games?
Solution: From part (a), the number of games that Cubs win out of the first 4 games is $Binomial(4, p)$, hence

$$E[\text{number of games that Cubs will win out of the first 4 games}] = 4p$$

(c) Obtain the probability $P\{G = 6, \text{Cubs win the series}\}$.
Solution: Let $W_C = \{\text{Cubs win the series}\}$. For event $\{G = 6, W_C\}$ to occur we need Cubs to win 3 out of the first 5 games, and Cubs must win the 6th game. The number of games that Cubs win out of the first 5 games is $Binomial(5, p)$, and p is the probability that Cubs win game 6 if it is reached. Hence,

$$P\{G = 6, W_C\} = \left[\binom{5}{3} p^3(1-p)^2\right] p = \binom{5}{3} p^4(1-p)^2$$

(d) Obtain $p_G(n)$, the pmf of G, for all n.
Solution: Clearly $G \in \{4, 5, 6, 7\}$. Using total probability, and following reasoning similar to part (b), for $n \in \{4, 5, 6, 7\}$:

$$\begin{aligned}
p_G(n) &= P\{G = n\} = P\{G = n, \text{Cubs win}\} + P\{G = n, \text{Cubs do not win}\} \\
&= \left[\binom{n-1}{3} p^3(1-p)^{n-1-3}\right] p + \left[\binom{n-1}{3} (1-p)^3 p^{n-1-3}\right] (1-p) \\
&= \binom{n-1}{3} p^4(1-p)^{n-4} + \binom{n-1}{3} (1-p)^4 p^{n-4}
\end{aligned}$$

4. **[14 points]** Suppose S and T represent the lifetimes of two phones, the lifetimes are independent, and each has the exponential distribution with parameter $\lambda = 1$.

(a) Obtain $P\{|S - T| \leq 1\}$.
Solution: $P\{|S - T| \leq 1\} = \int\int_R e^{-u}e^{-v}dudv$, where R is the infinite strip in the positive quadrant defined by $R = \{u \geq 0, v \geq 0, |u - v| \leq 1\}$. The complement of R in the positive quadrant is the union of the region $S_1 = \{u \geq 1, 0 \leq v \leq u - 1\}$ below R, and a similar region, S_2, above R. By symmetry, $P\{(S, T) \in S_1\} = P\{(S, T) \in S_2\}$ so that $P\{|S - T| \leq 1\} = 1 - 2P\{(S, T) \in S_1\}$. Since

$$\begin{aligned}
P\{(S, T) \in S_1\} &= \int_0^\infty \int_{v+1}^\infty e^{-u-v} dudv \\
&= \int_0^\infty e^{-v} \int_{v+1}^\infty e^{-u} dudv \\
&= \int_1^\infty e^{-2v-1} dv = \frac{e^{-1}}{2},
\end{aligned}$$

it follows that $P\{|S - T| \leq 1\} = 1 - e^{-1}$.
ALTERNATIVELY, $|S - T|$ is the remaining lifetime of the other phone, after one phone fails. By the memoryless property of the exponential distribution, it follows that $|S - T|$ has the same distribution as S or T. So $P\{|S - T| \leq 1\} = 1 - e^{-1}$.

(b) Let $Z = (S-1)^2$. Obtain $f_Z(c)$, the pdf of Z, for all c.

Solution: Clearly $P\{Z \geq 0\} = 1$. For $c \geq 0$, $F_Z(c) = P\{(S-1)^2 \leq c\} = P\{-\sqrt{c} \leq S-1 \leq \sqrt{c}\} = P\{1-\sqrt{c} \leq S \leq 1+\sqrt{c}\}$. So

$$F_Z(c) = \begin{cases} 0 & c < 0 \\ \int_{1-\sqrt{c}}^{1+\sqrt{c}} e^{-u}du = e^{-1+\sqrt{c}} - e^{-1-\sqrt{c}} & 0 \leq c < 1 \\ \int_0^{1+\sqrt{c}} e^{-u}du = 1 - e^{-1-\sqrt{c}} & c \geq 1 \end{cases}$$

Differentiating with respect to c yields

$$f_Z(c) = \begin{cases} 0 & c < 0 \\ \frac{e^{-1+\sqrt{c}}+e^{-1-\sqrt{c}}}{2\sqrt{c}} & 0 \leq c < 1 \\ \frac{e^{-1-\sqrt{c}}}{2\sqrt{c}} & c \geq 1 \end{cases}$$

5. **[20 points]** Assume power surges occur as a Poisson process with rate 3 per hour. These events cause damage to a certain system (say, a computer).

(a) Obtain $F_{T_3}(t)$, the CDF of the time when the third power surge occurs, for all $t \geq 0$, measured for some reference time 0. NOTE: Give a simple answer that does not involve an integral or the sum of an infinite series. (*Hint:* It might be easier to first obtain the complementary CDF.)

Solution: The third surge takes place by time t if and only if at least three surges occur by time t. That is, $T_3 \leq t$ if and only if $N_t \geq 3$. Thus,
$P\{T_3 \leq t\} = P\{N_t \geq 3\} = 1 - P\{N_t \leq 2\} = 1 - (1 + \lambda t + \frac{(\lambda t)^2}{2})e^{-\lambda t}$, where $\lambda = \frac{1}{6}$.

(b) Assume that a single power surge occurring in a certain 10 minute period will cause the system to crash. What is the probability that the system will crash in that period?

Solution: The rate of power surges is $\lambda = 3$ per hour. The duration of the service period, t_o, is 10 minutes, or $t_o = 1/6$ hour, and $\lambda t_o = 1/2$. Let the number of power surges in 10 minutes be N.

$$P\{N \geq 1\} = 1 - P\{N = 0\} = 1 - e^{-1/2}.$$

(c) Obtain
$P\{$exactly 1 power surge during 1-3pm AND exactly 2 power surges during 2-6pm$\}$.

Solution: The two time intervals overlap, so we need to look at the time intervals $I_1 = [1,2]$, $I_2 = (2,3]$, and $I_3 = (3,6]$. We want to find the probability of one power surge during $I_1 \cup I_2$ and two power surges during $I_2 \cup I_3$. There are two mutually exclusive ways for this to happen:
(one surge in I_1, no surges in I_2, two surges in I_3) or
(no surge in I_1, one surge in I_2, one surge in I_3).

These two events have probabilities $(\lambda e^{-\lambda})(e^{-\lambda})(\frac{(3\lambda)^2 e^{-3\lambda}}{2})$ and $(e^{-\lambda})(\lambda e^{-\lambda})(3\lambda e^{-3\lambda})$, respectively. Adding these gives the total probability, $\left[\frac{9\lambda^3}{2} + 3\lambda^2\right] e^{-5\lambda} = (148.5)e^{-15}$.

6. **[22 points]** Let (X, Y) be uniformly distributed over the triangular region with vertices $(0, 0)$, $(1/2, 2)$, and $(1, 0)$.

   (a) Obtain $f_{X,Y}(u, v)$, the joint pdf of X and Y, for all u and v.
   Solution: The triangle has height 2 and base one, so it has unit area, so the joint pdf is one inside the triangle and zero outside. That is,

$$f_{X,Y}(u, v) = \begin{cases} 1 & (u, v) \in T \\ 0 & \text{otherwise} \end{cases}$$

   where $T = \{(u, v); 0 \le u \le 1, 0 \le v \le \min(4u, 4 - 4u)\}$, or equivalently, $T = \{(u, v) : 0 \le v \le 2, \frac{v}{4} \le u \le 1 - \frac{v}{4}\}$.

   (b) Obtain $f_Y(v)$, the marginal pdf of Y, for all v.
   Solution: For $v \ge 2$ or $v < 0$, $f_Y(v) = 0$. For $0 \le v < 2$,

$$\begin{aligned} f_Y(v) &= \int_{-\infty}^{\infty} f_{X,Y}(u, v) du = \int_{\frac{v}{4}}^{1-\frac{v}{4}} 1 du \\ &= 1 - \frac{v}{2} \end{aligned}$$

   (c) Obtain $f_{X|Y}(u|v)$, the conditional pdf of X given Y, for all u and v.
   Solution: For $0 \le v < 2$,

$$f_{X|Y}(u|v) = \frac{f_{X,Y}(u, v)}{f_Y(v)} = \begin{cases} \frac{2}{2-v} & \frac{v}{4} < u < 1 - \frac{v}{4} \\ 0 & \text{else} \end{cases}$$

   That is, given $Y = v$, the conditional distribution of X is uniform over the interval $\left[\frac{v}{4}, 1 - \frac{v}{4}\right]$. For $v < 0$ or $v \ge 2$, the conditional pdf $f_{X|Y}(u|v)$ is not defined.

   (d) Obtain $E[X|Y = v]$ for all v.
   Solution: The mean of the uniform distribution over $\left[\frac{v}{4}, 1 - \frac{v}{4}\right]$ is the midpoint of the interval, or $\frac{1}{2}$. Thus, for $0 \le v < 2$, $E[X|Y = v] = \frac{1}{2}$. (For other v, $E[X|Y = v]$ is not defined.) Another way to get this result is to use the formulas:

$$\begin{aligned} E[X|Y = v] &= \int_{-\infty}^{\infty} u f_{X|Y}(u|v) du \\ &= \int_{\frac{v}{4}}^{1-\frac{v}{4}} \frac{2u}{2 - v} du \\ &= \frac{1}{2 - v} u^2 \Big|_{\frac{v}{4}}^{1-\frac{v}{4}} = \frac{1}{2} \end{aligned}$$

(e) Determine if X and Y are independent and indicate why or why not.
Solution: X and Y are not independent because the support is not a product set. Another reason is that $f_{X|Y}(u|v)$ depends on v.

7. [**18 points**] Consider an On-Off Keying (OOK) comunication system, where we either transmit $x = 0$ or $x = A$ with $A > 0$ being a constant. At the receiver side, detecting if a "0" was transmitted ($x = 0$) or a "1" was transmitted ($x = A$) can be posed as the following binary hypothesis testing problem for observation Y:

$$\mathcal{H}_0 : \ Y = W \qquad\qquad \mathcal{H}_1 : \ Y = A + W$$

where W is a $\mathcal{N}(0, \sigma^2)$ random variable corresponding to additive noise at the receiver.

(a) Determine $f_0(y)$, the pdf of Y under $\mathcal{H}_0$, and also $f_1(y)$, the pdf of Y under $\mathcal{H}_1$.
Solution: For $\mathcal{H}_0$, $Y = W$ hence

$$f_0(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}.$$

For $\mathcal{H}_1$, $Y = A + W$. Since Y is obtained from W by adding the constant A, the pdf of Y is obtained by shifting the pdf of W to the right by A. That is, under $\mathcal{H}_1$, Y has the $\mathcal{N}(A, \sigma^2)$ distribution:

$$f_1(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-A)^2}{2\sigma^2}}.$$

(b) Determine the MAP decision rule assuming the priors π_0 and π_1 are known. Express the rule in terms of Y in the simplest way possible.
Solution: The likelihood ratio test for the MAP rule is:

$$\frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y-A)^2}}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}y^2}} > \frac{\pi_0}{\pi_1}.$$

Cancelling common factors and taking the logarithm to both sides yields:

$$-\frac{1}{2\sigma^2}\left(-2Ay + A^2\right) > \ln\left(\frac{\pi_0}{\pi_1}\right).$$

Hence, the MAP rule decides $\mathcal{H}_1$ if $Y > \frac{\sigma^2}{A}\ln\left(\frac{\pi_0}{\pi_1}\right) + \frac{A}{2}$ and $\mathcal{H}_0$ otherwise.

(c) Assume that $\pi_0 = \pi_1$. Determine the average error probability, p_e. You can leave your answer in terms of the Q or the Φ functions.
Solution: If $\pi_0 = \pi_1$, the MAP rule decides $\mathcal{H}_1$ if $Y > \frac{A}{2}$ and $\mathcal{H}_0$ otherwise.

$$p_{FA} = P(\text{decide } \mathcal{H}_1 | \mathcal{H}_0 \text{ is true}) = P\left(y > \frac{A}{2} | \mathcal{H}_0\right) = Q\left(\frac{\frac{A}{2}}{\sigma}\right) = Q\left(\frac{A}{2\sigma}\right).$$

$$p_{miss} = P(\text{decide}\mathcal{H}_0 | \mathcal{H}_1 \text{is true}) = P\left(y \leq \frac{A}{2} | \mathcal{H}_1\right) = \Phi\left(\frac{\frac{A}{2} - A}{\sigma}\right) = \Phi\left(-\frac{A}{2\sigma}\right) = Q\left(\frac{A}{2\sigma}\right).$$

Thus,

$$p_e = \pi_0 p_{FA} + \pi_1 p_{miss} = \frac{1}{2}Q\left(\frac{A}{2\sigma}\right) + \frac{1}{2}Q\left(\frac{A}{2\sigma}\right) = Q\left(\frac{A}{2\sigma}\right).$$

8. **[18 points]** Suppose X and Y are *zero-mean unit-variance* jointly Gaussian random variables with correlation coefficient $\rho = 0.5$.

   (a) Obtain $Var(3X - 2Y)$.
   Solution: $Var(3X - 2Y) = 3^2 \cdot Var(X) + 2^2 \cdot Var(Y) - 2 \cdot 3 \cdot 2 \cdot \mathsf{cov}(X,Y) = 9 + 4 - 12 \times \frac{1}{2} = 7$.

   (b) Obtain $P\{(3X - 2Y)^2 \leq 28\}$ in terms of the Q or the Φ functions.
   Solution: $\mathsf{E}[3X - 2Y] = 3 \cdot \mathsf{E}[X] - 2 \cdot \mathsf{E}[Y] = 0$. Furthermore, since X and Y are *jointly Gaussian* random variables, $3X - 2Y$ is also a Gaussian random variable, and we have that
   $P\{(3X-2Y)^2 \leq 28\} = P\{-\sqrt{28} \leq 3X - 2Y \leq \sqrt{28}) = \Phi\Bigg(\dfrac{\sqrt{28}-0}{\sqrt{7}}\Bigg) - \Phi\Bigg(-\dfrac{\sqrt{28}-0}{\sqrt{7}}\Bigg)$
   $= \Phi(2) - \Phi(-2) = \Phi(2) - [1 - \Phi(2)] = 2\Phi(2) - 1$.

   (c) Obtain $\mathsf{E}[Y \mid X = 3]$.
   Solution: Since X and Y are *jointly Gaussian* random variables, the conditional mean of Y given $X = \alpha$ is the same as the *linear* MMSE estimator of Y given $X = \alpha$, *viz.* $\mu_{\mathbb{Y}} + \rho(\sigma_Y/\sigma_X)(\alpha - \mu_X)$
   $= 0 + 0.5 \times 1 \times (3 - 0) = 3/2$.

9. **[12 points]** Observations $X_1, \ldots, X_T$ produced by a drone's altimeter are assumed to have the form $X_t = bt + W_t$ where b is an unknown constant representing the rate of ascent of the drone (if $b < 0$ it means the drone is descending) and $W_1, \ldots, W_T$ represent observation noise and are assumed to be independent, $N(0,1)$ random variables.

   (a) Write down the joint pdf of $X_1, \ldots, X_T$.
   Solution: X_t is $N(bt, 1)$ so $f_{X_t}(x_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_t - bt)^2}{2}}$. Since the observations are independent, the joint pdf is the product of the marginal pdfs:

$$f_{X_1,\ldots,X_T}(x_1, \ldots, x_T) = \frac{1}{(2\pi)^{T/2}} e^{-\sum_{t=1}^{T} \frac{(x_t - bt)^2}{2}}$$

   (b) Obtain the maximum likelihood estimator of b for a particular vector of observations $x_1, \ldots, x_T$.
   Solution: $\widehat{b}_{ML}$ is the value of b that maximizes $f_{X_1,\ldots,X_T}(x_1, \ldots, x_T)$, or equivalently, minimizes $\sum_{t=1}^{T} \frac{(x_t - bt)^2}{2}$. This is a quadratic function of b that is minimized by setting the derivative to zero.

$$\frac{d()}{db} = \sum_{t=1}^{T} (x_t - bt)(-t) = b \sum_{t=1}^{T} t^2 - \sum_{t=1}^{T} x_t t$$

Setting the derivative to zero yields

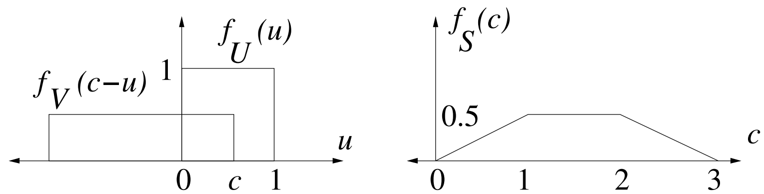
$$\widehat{b}_{ML} = \frac{\sum_{t=1}^{T} x_t t}{\sum_{t=1}^{T} t^2}.$$

10. [**18 points**] Suppose U and V are independent random variables such that U is uniformly distributed over $[0,1]$ and V is uniformly distributed over $[0,2]$. Let $S = U + V$.

   (a) Obtain the mean and variance of S.
   Solution: $E[S] = E[U] + E[V] = 0.5 + 1 = 1.5$. $\mathrm{Var}(S) = \mathrm{Var}(U) + \mathrm{Var}(V) = \frac{1}{12} + \frac{2^2}{12} = \frac{5}{12}$.

   (b) Derive and carefully sketch the pdf of S.
   Solution:

$$f_S(c) = \int_{-\infty}^{\infty} f_U(u) f_V(c-u) du = \begin{cases} c/2 & 0 \le c \le 1 \\ 1/2 & 1 \le c \le 2 \\ (c-2)/2 & 2 \le c \le 3 \\ 0 & \text{else} \end{cases}$$



   (c) Obtain $\widehat{E}[U|S]$, the minimum mean square error linear estimator of U given S.
   Solution: $\mathrm{Cov}(U,S) = \mathrm{Cov}(U, U+V) = \mathrm{Var}(U) = \frac{1}{12}$. Thus,

$$\widehat{E}[U|S] = E[U] + \frac{\mathrm{Cov}(U,S)}{\mathrm{Var}(S)}(S - E[S]) = \frac{1}{2} + \frac{1}{5}(S - 1.5)$$

11. [**30 points**] (3 points per answer)
In order to discourage guessing, 3 points will be deducted for each incorrect answer (no penalty or gain for blank answers). A net negative score will reduce your total exam score.

   (a) Suppose X and Y are jointly continuous-type random variables with finite variance.

TRUE	FALSE	
☐	☐	If the MMSE for estimating Y from X is $\mathrm{Var}(Y)$, then X and Y must be uncorrelated.
☐	☐	If X and Y are uncorrelated then the MMSE for estimating Y from X is $\mathrm{Var}(Y)$.
☐	☐	If X and Y are uncorrelated and jointly Gaussian, then the MMSE for estimating Y from X is $\mathrm{Var}(Y)$.

   Solution: True, False, True

(b) Let $X_1, \ldots, X_m$ be independent random variables, each with the binomial distribution with parameters 10 and p, where $0 < p < 1$, and let $S_m = X_1 + \ldots + X_m$.

TRUE FALSE

☐ ☐ S_m has a binomial distribution

☐ ☐ $\lim_{m\to\infty} P\left\{\frac{S_m}{m} \geq 10p(1-p)\right\} = 1$

Solution: True,True

(c) Consider a binary hypothesis testing problem. Let the subscript ML denote the maximum likelihood rule, and subscript MAP denote the maximum a posteriori rule.

TRUE FALSE

☐ ☐ It is possible that $p_{\text{miss},ML} < p_{\text{miss},MAP}$.

☐ ☐ It is possible that $p_{\text{false alarm},ML} = p_{\text{false alarm},MAP}$.

☐ ☐ If $\pi_0 > \pi_1$ it is possible that $p_{\text{miss},ML} < p_{\text{miss},MAP}$.

Solution: True, True, True

(d) Let X and Y be uncorrelated, jointly Gaussian random variables, with parameters μ_X, μ_Y, σ_X^2 and σ_Y^2.

TRUE FALSE

☐ ☐ $f_{XY}(u,v) = f_X(u)f_Y(v)$ for all real u, v

☐ ☐ $E[XY] = 0$.

Solution: True,False