University of Illinois Fall 2016

ECE 313: Conflict Final Exam

Tuesday, December 13, 2016
7 p.m. — 10 p.m.
ECEB 1013

1. **[14 points]** A drawer contains 4 black, 6 red, and 8 yellow socks. Two socks are selected at random from the drawer.

   (a) What is the probability the two socks are of the same color?

   Solution: Let B, R, and Y denote the sets of black, red, and yellow socks, with cardinalities 4, 6, and 8, respectively. A suitable choice of sample space for this experiment is
   $$\Omega = \{S : |S| = 2 \text{ and } S \subset B \cup R \cup Y\},$$
   where S represents the set of two socks selected. The cardinality of Ω is
   $$|\Omega| = \binom{4+6+8}{2} = \binom{18}{2} = 153.$$
   Let F be the event $F = \{S : |S| = 2 \text{ and } S \subset B \text{ or } S \subset R \text{ or } S \subset Y\}$. Then,
   $$|F| = \binom{4}{2} + \binom{6}{2} + \binom{8}{2} = 6 + 15 + 28 = 49.$$
   Thus,
   $$P(F) = \frac{49}{153}.$$

   (b) What is the conditional probability both socks are yellow given they are of the same color?

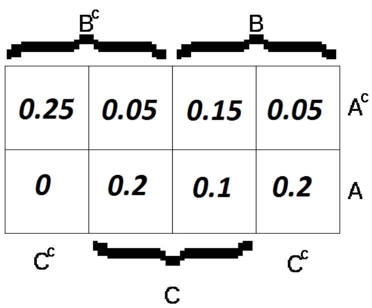
   Solution: Let $G = \{S : |S| = 2 \text{ and } S \subset Y\}$. Note that $G \subset F$ and $|G| = \binom{8}{2} = 28$. Therefore,
   $$P(G|F) = \frac{P(FG)}{P(F)} = \frac{P(G)}{P(F)} = \frac{28}{49} = \frac{4}{7}.$$

2. **[14 points]** The two parts of this problem are unrelated.

   (a) Suppose A, B, and C are events for a probability experiment such that B and C are mutually independent, $P(A) = P(B^c) = P(C) = 0.5$, $P(AB) = P(AC) = 0.3$, and $P(ABC) = 0.1$. Fill in the probabilities of all events in a Karnaugh map. Show your work but use the map on the right to depict your final answer.

   Solution: Start filling $P(ABC) = 0.1$.
   Then use $P(CA) = 0.3$ to get $P(CAB^c) = P(CA) - P(CAB) = 0.3 - 0.1 = 0.2$.
   Similarly, use $P(BA) = 0.3$ to get $P(BAC^c) = P(BA) - P(BAC) = 0.3 - 0.1 =$

0.2.
Then, use independence of C and B to get $P(CBA^c) = P(CB) - P(BAC) = P(C)P(B) - P(BAC) = 0.5^2 - 0.1 = 0.15$.
Then, use $P(B) = 0.5$ to get $P(C^cBA^c) = P(B) - P(BAC) - P(BAC^c) - P(CBA^c) = 0.5 - 0.1 - 0.2 - 0.15 = 0.05$.
Similarly, use $P(C) = 0.5$ to get $P(CB^cA^c) = P(C) - P(CAB^c) - P(CBA) - P(CBA^c) = 0.5 - 0.2 - 0.1 - 0.15 = 0.05$.
Then, use $P(A) = 0.5$ to get $P(AB^cC^c) = P(A) - P(CAB^c) - P(CBA) - P(CAB^c) = 0.5 - 0.2 - 0.1 - 0.2 = 0$.
Finally, the remaining probability to add up to one is $P(C^cB^cA^c) = 0.25$.



(b) Let A, B be two disjoint events on a sample space Ω. Obtain a formula for the probability of A occurring before B in an infinite sequence of independent trials.
Solution: A and B occur with probabilities $P(A)$ and $P(B)$, respectively. Consider the first trial:
First Trial: Either A occurs or B occurs or neither A nor B occurs.

- If A occurs, then the probability that A occurs before B is 1.
- If B occurs, then the probability that A occurs before B is 0.
- If neither A nor B occurs, then the process starts over.

Let s be the probability that neither A nor B occurs at any independent trial. Then $s = 1 - P(A) - P(B)$ due to $A \cap B = \emptyset$. Therefore,

$$\begin{aligned} P(A \text{ before } B) &= P(A) + sP(A) + s^2P(A) + \cdots + s^nP(A) + \cdots \\ &= P(A)\sum_{n=0}^{\infty} s^n = P(A)\frac{1}{1-s} = \frac{P(A)}{P(A)+P(B)}. \end{aligned}$$

Alternatively: Let s be the probability that neither A nor B occurs in a given independent trial. If neither A nor B occurs on the first trial, then the process starts over. So $P(A \text{ before } B) = P(A) + sP(A \text{ before } B)$. Solving this equation for $P(A \text{ before } B)$ yields $P(A \text{ before } B) = \frac{P(A)}{1-s} = \frac{P(A)}{P(A)+P(B)}$.

3. **[20 points]** Bob performs an experiment comprising a series of independent trials. On each trial, he simultaneously flips a set of three fair coins.

(a) Given that Bob has just had a trial with all 3 coins landing on tails, what is the probability that both of the next two trials will also have this result?
Solution: Since the trials are independent, the given information is irrelevant. $P(\text{next 2 trials result in 3 tails}) = (1/8)^2 = 1/64$.

(b) Whenever all three coins land on the same side in any given trial, Bob calls the trial a success. Obtain the pmf for K, the number of trials up to, but not including, the second success.

Solution: The probability that 1 success comes in the first k trials, where the next trial will result in the second success, can be expressed as:

$$p_K(k) = \binom{k}{1}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{k-1}$$

(c) Alice conducts an experiment like Bob's, except that she uses 4 coins for the first trial, and then she obeys the following rule: Whenever all of the coins land on the same side in a trial, Alice permanently removes one coin from the experiment and continues with the trials. She follows this rule until the third time she removes a coin, at which point the experiment ceases. Obtain $E[N]$, where N is the number of trials in Alices experiment.

Solution: N the number of trials in Alice's experiment, can be expressed as the sum of 3 independent random variables, X, Y, and Z. X is the number of trials until Alice removes the first coin, Y the number of additional trials until she removes the second coin, and Z the additional number until she removes the third coin. We see that X is a geometric random variable with parmeter 1/8, Y is geometric with parameter 1/4, and Z geometric with parameter 1/2. Hence,

$$E[N] = E[X] + E[Y] + E[Z] = 8 + 4 + 2 = 14.$$

4. **[14 points]** Suppose S and T represent the lifetimes of two phones, the lifetimes are independent, and each has the exponential distribution with parameter $\lambda = 1$.

(a) Obtain $P\{|S-T| \leq 1\}$.

Solution: $P\{|S-T| \leq 1\} = \int\int_R e^{-u}e^{-v}dudv$, where R is the infinite strip in the positive quadrant defined by $R = \{u \geq 0, v \geq 0, |u-v| \leq 1\}$. The complement of R in the positive quadrant is the union of the region $S_1 = \{u \geq 1, 0 \leq v \leq u-1\}$ below R, and a similar region, S_2, above R. By symmetry, $P\{(S,T) \in S_1\} = P\{(S,T) \in S_2\}$ so that $P\{|S-T| \leq 1\} = 1 - 2P\{(S,T) \in S_1\}$. Since

$$\begin{aligned} P\{(S,T) \in S_1\} &= \int_0^\infty \int_{v+1}^\infty e^{-u-v}dudv \\ &= \int_0^\infty e^{-v} \int_{v+1}^\infty e^{-u}dudv \\ &= \int_1^\infty e^{-2v-1}dv = \frac{e^{-1}}{2}, \end{aligned}$$

it follows that $P\{|S-T| \leq 1\} = 1 - e^{-1}$.

ALTERNATIVELY, $|S-T|$ is the remaining lifetime of the other phone, after one phone fails. By the memoryless property of the exponential distribution, it follows that $|S-T|$ has the same distribution as S or T. So $P\{|S-T| \leq 1\} = 1 - e^{-1}$.

(b) Let $Z = (S-1)^2$. Obtain $f_Z(c)$, the pdf of Z, for all c.

Solution: Clearly $P\{Z \geq 0\} = 1$. For $c \geq 0$, $F_Z(c) = P\{(S-1)^2 \leq c\} = P\{-\sqrt{c} \leq S-1 \leq \sqrt{c}\} = P\{1-\sqrt{c} \leq S \leq 1+\sqrt{c}\}$. So

$$F_Z(c) = \begin{cases} 0 & c < 0 \\ \int_{1-\sqrt{c}}^{1+\sqrt{c}} e^{-u} du = e^{-1+\sqrt{c}} - e^{-1-\sqrt{c}} & 0 \leq c < 1 \\ \int_0^{1+\sqrt{c}} e^{-u} du = 1 - e^{-1-\sqrt{c}} & c \geq 1 \end{cases}$$

Differentiating with respect to c yields

$$f_Z(c) = \begin{cases} 0 & c < 0 \\ \frac{e^{-1+\sqrt{c}} + e^{-1-\sqrt{c}}}{2\sqrt{c}} & 0 \leq c < 1 \\ \frac{e^{-1-\sqrt{c}}}{2\sqrt{c}} & c \geq 1 \end{cases}$$

5. **[20 points]** An X-ray source transmits photons towards a detector at mean rate $\lambda = 8$ photons/msec according to a Poisson process. Suppose each photon transmitted is detected independently with probability $p = 0.25$.

(a) What is the pdf of the time the fifth photon is transmitted (measured in msec)?

Solution: The distribution of the r^{th} arrival has the Erlang distribution with parameters λ and r. Taking $r = 5$ and $\lambda = 8$ gives the pdf: $f(t) = \frac{\lambda^5 t^4}{24} e^{-8t}$ for $t \geq 0$.

(b) Given n photons are transmitted in a certain period, what is the probability k photons are detected. (Assume $n \geq 1$ and $0 \leq k \leq n$.)

Solution: Each photon transmitted represents an independent trial, with probability of success (i.e. detection) equal to p. Thus, the number detected has the binomial distribution with parameters n and p. So $P\{k \text{ photons detected}\} = \binom{n}{k}(0.25)^k(0.75)^{n-k}$.

(c) What is the pmf for the number of photons *detected* in an interval of duration 10 msec? Simplify your answer as much as possible. (Hint: Let X denote the number of photons transmitted and Y the number detected. Obtain the pmf of Y using the law of total probability.)

Solution: In the notation of the hint, Y has the Poisson distribution with parameter $10\lambda = 80$, and, given $Y = n$, X has the binomial distribution with parameters n and p. Thus, $p_{Y,X}(n,k) = \frac{(80)^n e^{-80}}{n!}\binom{n}{k}p^k(1-p)^{n-k}$. To find the marginal pmf, p_Y, we sum over n:

$$\begin{aligned} p_Y(k) &= \sum_{n=k}^{\infty} \frac{(80)^n e^{-80}}{n!} \binom{n}{k} p^k (1-p)^{n-k} \\ &= \frac{p^k 80^k e^{-80}}{k!} \sum_{n=k}^{\infty} \frac{(80(1-p))^{n-k}}{(n-k)!} \\ &= \frac{(80p)^k e^{-80}}{k!} e^{80(1-p)} = \frac{e^{-20}(20)^k}{k!} \end{aligned}$$

That is, the number of photons detected has the Poisson distribution with parameter (i.e. mean) 20. (Note: So independent subsampling maps one Poisson distribution to another. The same property is easily seen for the binomial distribution, and therefore also for the Poisson distribution.)

6. **[22 points]** Suppose that a point (X, Y) is picked uniformly in the triangle $\{(x, y)|0 \leq x \leq 1, 0 \leq y \leq x\}$.

   (a) Obtain $f_{X,Y}(u, v)$, the joint pdf of X and Y, for all u and v.
   Solution: The triangle has base one and height one, so it has area of $\frac{1}{2}$. Hence, the joint pdf is two inside the triangle and zero outside. That is,

   $$f_{X,Y}(u,v) = \begin{cases} 2 & 0 \leq u \leq 1, 0 \leq v \leq u \\ 0 & \text{otherwise} \end{cases}$$

   (b) Obtain $f_{Y|X}(v|u)$, the conditional pdf of Y given X, for all u and v.
   Solution: The conditional pdf $f_{Y|X}(v|u)$ will be constant because the joint pdf is constant, and for each fixed $u \in (0, 1)$, $0 \leq v \leq u$. Hence, $f_{Y|X}(v|u) = \frac{1}{u}$ for $u \in (0, 1)$ and $0 \leq v \leq u$. The conditional pdf $f_{Y|X}(v|u)$ is not defined for $u \notin (0, 1)$ because the marginal of X is zero there.

   (c) Calculate $E[Y|X = u]$.
   Solution: From part (b), given $X = u \in (0, 1)$, Y is $\sim Uniform[0, u]$. Thus,

   $$E[Y|X = u] = \frac{u}{2}.$$

   (d) Compute $E[(X - Y)^2|X = u]$.
   Solution: From part (b), given $X = u \in (0, 1)$, Y is $\sim Uniform[0, u]$. Thus,

   $$E[(X-Y)^2|X = u] = \int_0^u (u-y)^2 f_{Y|X}(v|u) dv = \int_0^u (u-y)^2 \frac{1}{u} dv = \frac{1}{u} \int_0^u v^2 dv = \frac{u^2}{3}.$$

7. **[18 points]** Suppose two different data streams, S_1 and S_2, share a communication channel. S_1 transmits on any given day with probability $\frac{2}{3}$, while S_2 transmits on the other days. Let X be the number of bits per hour sent over the channel . If S_1 transmits, X is a geometric random variable with parameter $p = \frac{2}{3}$, whereas if S_2 transmits, X is a geometric random variable with parameter $p = \frac{1}{2}$. You have no way to find out who is is trasnmitting except by observing the number of bits sent.

(a) Suppose you use the following rule to decide who is trasnmitting today, based on observation of the data rate for one hour, X:

- If $X > 1$, say S_1
- If $X = 1$, say S_2

What is the probability that this rule makes an error?

Solution: Let $\pi_1 = P\{S_1 \text{ transmits}\} = \frac{2}{3}$, and let $\pi_2 = P\{S_2 \text{ transmits}\} = 1-\pi_1-\frac{1}{3}$. Let $p_1(k)$ be the probability that $X = k$ if S_1 transmits, so $p_1(k) = \frac{1}{3}^{k-1}\frac{2}{3}$. Let $p_2(k)$ be the probability that $X = k$ if S_2 transmits, so $p_2(k) = \frac{1}{2}^{k-1}\frac{1}{2} = \frac{1}{2}^{k}$. Then

$$\begin{aligned}
P\{\text{error}\} &= P\{\text{error}|S_1 \text{ transmits}\}P\{S_1 \text{ transmits}\} + P\{\text{error}|S_2 \text{ transmits}\}P\{S_2 \text{ transmits}\} \\
&= P\{X = 1|S_1 \text{ transmits}\}\pi_1 + P\{X > 1|S_2 \text{ transmits}\}\pi_2 = p_1(1)\pi_1 + [1 - p_2(1)]\pi_2 \\
&= \frac{2}{3}\frac{2}{3} + \frac{1}{2}\frac{1}{3} = \frac{11}{18}
\end{aligned}$$

(b) The maximum likelihood decision rule for this hypothesis testing problem can be stated as:

- If $X \geq k_{ML}$, say S_2 transmits.
- Otherwise, say S_1 transmits.

for some positive integer value k_{ML}. Obtain the value of k_{ML}.

Solution: The maximum likelihood decision rule is to say S_2 transmits if X takes on a value k such that:

$$1 < \frac{p_2(k)}{p_1(k)} = \frac{\frac{1}{2}^{k}}{\frac{2}{3}\frac{1}{3}^{k-1}} = \frac{1}{2}\left(\frac{3}{2}\right)^{k}$$

The likelihood ratio $\frac{1}{2}\left(\frac{3}{2}\right)^{k}$ is strictly increasing in k and its values for $k = 1, 2$ are $\frac{3}{4}, \frac{9}{8}$, respectively, So $k_{ML} = 2$.

8. [**18 points**] Suppose that X, Y are jointly Gaussian random variables with $\mu_X = \mu_Y = 0$ and $\sigma_X = \sigma_Y = 1$. Let their correlation coefficient be ρ with $|\rho| < 1$. Based on (X, Y), we define the following random variables:

$$W = \left(\frac{1}{2\sqrt{1+\rho}} + \frac{1}{2\sqrt{1-\rho}}\right)X + \left(\frac{1}{2\sqrt{1+\rho}} - \frac{1}{2\sqrt{1-\rho}}\right)Y$$
$$Z = \left(\frac{1}{2\sqrt{1+\rho}} - \frac{1}{2\sqrt{1-\rho}}\right)X + \left(\frac{1}{2\sqrt{1+\rho}} + \frac{1}{2\sqrt{1-\rho}}\right)Y$$

(a) Are W, Z jointly Gaussian? Justify your answer.

Solution: Since X, Y are jointly Gaussian, every linear combination $aX + bY$ is Gaussian. Every linear combination of W, Z corresponds to a linear combination of X, Y. Therefore, it will be Gaussian. This implies that W, Z are jointly Gaussian.

(b) Obtain $f_{W,Z}(w,z)$.

Solution: Clearly, $\mu_W = \mu_Z = 0$. For the variances, we have:

$$\sigma_W^2 = \mathrm{Cov}(W,W) = \frac{1-\rho^2}{(1+\rho)(1-\rho)} = 1$$

and similarly $\sigma_Z^2 = 1$. Finally, by performing the calculations,

$$\mathrm{Cov}(W,Z) = 0,$$

i.e., $\rho_{W,Z} = 0$. Therefore, W, Z are uncorrelated. Since they are jointly Gaussian, they are independent. This shows that

$$f_{WZ}(w,z) = f_W(w)f_Z(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{w^2}{2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}} = \frac{1}{2\pi}e^{-\frac{w^2+z^2}{2}},$$

for $w, z \in (-\infty, +\infty)$.

(c) Obtain the MMSE estimator of Z given W.

Solution: The MMSE estimator of Z given W is the conditional mean $E[Z|W]$. Since Z, W are independent, we have

$$E[Z|W] = E[Z] = \mu_Z = 0.$$

(d) Obtain the linear MMSE estimator of X given W.

Solution: We first compute $\mathrm{Cov}(X,W)$:

$$\begin{aligned}\mathrm{Cov}(X,W) &= \left(\frac{1}{2\sqrt{1+\rho}} + \frac{1}{2\sqrt{1-\rho}}\right)\mathrm{Var}(X) + \left(\frac{1}{2\sqrt{1+\rho}} - \frac{1}{2\sqrt{1-\rho}}\right)\mathrm{Cov}(X,Y) \\ &= \left(\frac{1}{2\sqrt{1+\rho}} + \frac{1}{2\sqrt{1-\rho}}\right) + \left(\frac{1}{2\sqrt{1+\rho}} - \frac{1}{2\sqrt{1-\rho}}\right)\rho \\ &= \frac{1}{2}\left(\sqrt{1+\rho} + \sqrt{1-\rho}\right).\end{aligned}$$

We now have:

$$\widehat{E}[X|W] = \mu_X + \frac{\mathrm{Cov}(X,W)}{\sigma_W^2}(W-\mu_W) = \frac{1}{2}\left(\sqrt{1+\rho}+\sqrt{1-\rho}\right)W.$$

9. **[12 points]** Let $U_1, \ldots, U_n$ be independent, exponentially distributed random variables with unknown parameter λ.

(a) Identify the ML estimator $\widehat{\lambda}$ for joint observations $U_1, \ldots, U_n$.

Solution: The likelihood is the product of the marginal distributions, because of the independence. Thus, $f_T(u_1, \ldots, u_n) = \lambda e^{-\lambda u_1} \cdots \lambda e^{-\lambda u_n} = \lambda^n e^{-\lambda s_n}$, where $s_n = u_1 + \cdots + u_n$. To find $\widehat{\lambda}_{ML}$ we maximize with respect to λ by (optionally taking log first) and setting derivative to zero. The result is $\widehat{\lambda}_{ML} = \frac{n}{s_n}$.

(b) Using the Chebychev inequality, identify a number of observations n large enough so that $[(0.9)\widehat{\lambda}_{ML}, (1.1)\widehat{\lambda}_{ML}]$ is a confidence interval for estimation of λ with confidence level 96%.

Solution: We need n large enough that $P\{(0.9)\widehat{\lambda}_{ML} \le \lambda \le (1.1)\widehat{\lambda}_{ML}\} \ge 0.96$. Equivalently, using $S_n = U_1 + \ldots + U_n$, we want $P\left\{\frac{(0.9)n}{S_n} \le \lambda \le \frac{(1.1)n}{S_n}\right\} \ge 0.96$, or $P\left\{0.9 \le \frac{\lambda S_n}{n} \le 1.1\right\} \ge 0.96$. To apply the Chebychev inequality we note that $E[\frac{\lambda S_n}{n}] = E[\lambda U_1] = 1$ and $\mathrm{Var}(\frac{\lambda S_n}{n}) = \frac{\lambda^2}{n^2}\mathrm{Var}(S_n) = \frac{\lambda^2 \mathrm{Var}(U_1)}{n} = \frac{1}{n}$, where we used the fact that the variance of the exponential distribution with parameter λ is $\frac{1}{\lambda^2}$. Thus, by the Chebychev inequality,

$$P\left\{\left|\frac{\lambda S_n}{n} - 1\right| \ge \delta\right\} \le \frac{1}{n\delta^2}.$$

Setting $\frac{1}{n\delta^2} = 0.04$ with $\delta = 0.1$ yields $n = \frac{1}{(0.1)^2(0.04)} = 2500$.

10. **[18 points]** Suppose U and V are independent random variables such that U is uniformly distributed over $[0,1]$ and V is uniformly distributed over $[0,2]$. Let $S = U + V$.

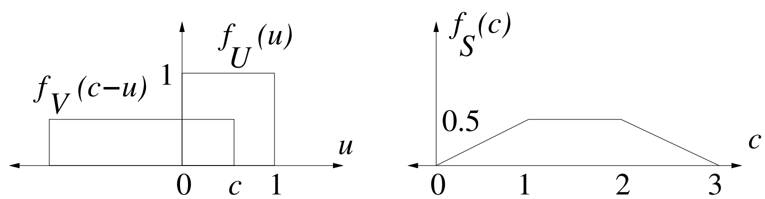
(a) Obtain the mean and variance of S.

Solution: $E[S] = E[U] + E[V] = 0.5 + 1 = 1.5$. $\mathrm{Var}(S) = \mathrm{Var}(U) + \mathrm{Var}(V) = \frac{1}{12} + \frac{2^2}{12} = \frac{5}{12}$.

(b) Derive and carefully sketch the pdf of S.

Solution:

$$f_S(c) = \int_{-\infty}^{\infty} f_U(u) f_V(c-u)\,du = \begin{cases} c/2 & 0 \le c \le 1 \\ 1/2 & 1 \le c \le 2 \\ (c-2)/2 & 2 \le c \le 3 \\ 0 & \text{else} \end{cases}$$



(c) Obtain $\widehat{E}[U|S]$, the minimum mean square error linear estimator of U given S.

Solution: $\mathrm{Cov}(U,S) = \mathrm{Cov}(U, U+V) = \mathrm{Var}(U) = \frac{1}{12}$. Thus,

$$\widehat{E}[U|S] = E[U] + \frac{\mathrm{Cov}(U,S)}{\mathrm{Var}(S)}(S - E[S]) = \frac{1}{2} + \frac{1}{5}(S - 1.5)$$

11. **[30 points]** (3 points per answer)
In order to discourage guessing, 3 points will be deducted for each incorrect answer (no penalty or gain for blank answers). A net negative score will reduce your total exam score.

(a) Suppose X and Y are jointly continuous-type random variables with finite variance.

TRUE	FALSE	
☐	☐	If the MMSE for estimating Y from X is $\mathrm{Var}(Y)$, then X and Y must be uncorrelated.
☐	☐	If X and Y are uncorrelated then the MMSE for estimating Y from X is $\mathrm{Var}(Y)$.

Solution: True, False

(b) Suppose Y is a nonnegative random variable with $E[Y] = 10$, and X is a random variable with mean 10 and variance 16.

TRUE	FALSE	
☐	☐	It is possible that the standard deviation of Y is 10.
☐	☐	It is possible that $P\{Y \geq 30\} = 1/4$.
☐	☐	It is possible that $P\{X \geq 0\} = 0.5$.

Solution: True, True, False

(c) Suppose X and Y are two Binomial random variables with parameters n_X, p_X, and n_Y, p_Y, respectively.

TRUE	FALSE	
☐	☐	If $Y = n_X - X$, then $p_y(k) = p_x(n_X - k)$.
☐	☐	If $n_X = n_Y > 20$ and $p_x(1) > p_y(1)$ then $E[X] > E[Y]$.
☐	☐	If $Z = X + Y$, then Z is $Binomial(n_X + n_Y, p_X + p_Y)$.

Solution: True, True, False

(d) Let A, B be nonempty events in a sample space Ω. Assume that $E_1, E_2, \ldots, E_n$ is a partition of Ω.

TRUE	FALSE	
☐	☐	If A, B are mutually exclusive, then $P(A\|B) = P(A)$.
☐	☐	Suppose that $A \neq B$. Then $\sum_{i=1}^{n} P(E_i\|A) \neq \sum_{i=1}^{n} P(E_i\|B)$.

Solution: False, False