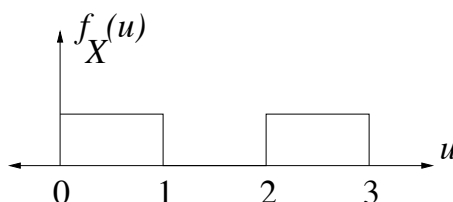# ECE 313: Hour Exam II

Wednesday, November 12, 2014
7:00 p.m. — 8:15 p.m.
Sect. B in 151 Everitt Lab, Sects. C&E in 100 Noyes Lab, Sect. D in 1404 Siebel Center

1. [**24 points**] (6 points) Suppose X is a random variable with the pdf shown.

   (a) (6 points) Carefully *sketch* and label the CDF of X.
   (b) (6 points) Carefully *sketch* and label the pdf of Y, where $Y = 2X + 1$.
   (c) (6 points) Find $E[X]$. Simplify as much as possible. Show your work or explain your reasoning.
   (d) (6 points) Find $P\{\ln(X) \geq 1\}$. (Hint: $e = \exp(1) \approx 2.72$.)

2. [**14 points**] Suppose that the lifetime of a battery produced by company EBunny can be modeled by a Gaussian random variable, with mean 300 days and variance 100 (days)^2.

   (a) (7 points) A buyer is interested in purchasing the batteries, but requires that at least 99% of the batteries delivered have lifespan of at least 275 days. Do the EBunny batteries meet the requirement? Justify your answer. (Note: A table of the Φ function is on the last page of the exam booklet.)
   (b) (7 points) Let Y be a binomial random variable with parameters n and p such that Y has the same expected value and variance as the Gaussian variable in the original problem statement. Find n and p.

3. [**12 points**] Let $\{N(t),\ t \geq 0\}$ be a Poisson process with rate λ.

   (a) (6 points) Express $E[N(t)N(t+s)]$, $s, t > 0$ as a function of λ, s, and t.
   (b) (6 points) Let $\lambda = 2$ arrivals/hour and assume that the Poisson process models the arrival of customers into a post office. Find the probability of the following event, which involves three conditions:
        (Three customers arrive between 1 and 3pm,
            one customer arrives between 2 and 3pm,
               and one customer arrives between 2 and 4pm.)

4. [**14 points**] Suppose X is a uniform random variable with support $[-1, 1]$ and $Y = X^4$.

   (a) (6 points) Find $E[Y]$.
   (b) (8 points) Find the pdf of Y, making sure to express it's value over the entire real line.

5. [**12 points**] The random variable X is assumed to have a Gaussian distribution. It is observed that $X = \sqrt{2}$.

   (a) (6 points) Suppose the variance of X is assumed to be one. Find the ML estimate of its mean given the observation.

(b) (6 points) (For this part we assume the mean is known and the variance is to be esti-mated, for the same observation $X = \sqrt{2}$.) Suppose the mean of X is assumed to be zero. Find the ML estimate of the variance of X given the observation.

6. **[24 points]** Suppose $f_{XY}(u, v) = \begin{cases} 3u & \text{if } 0 \leq u \leq 1, 2u \leq v \leq 2 \\ 0 & \text{else} \end{cases}$ with support shown:
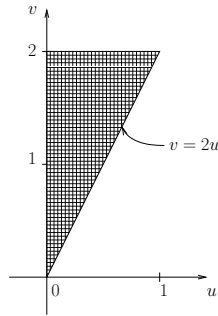
(a) (6 points) Are X and Y independent? Justify your answer.

(b) (6 points) Find $P\{Y > 2\sqrt{X}\}$.

(c) (6 points) Find the marginal pdf, $f_X(u)$, $-\infty < u < \infty$.

(d) (6 points) Find the conditional pdf $f_{Y|X}(v|u)$ for $0 < u < 1$ and $-\infty < v < \infty$.

Table 1: Φ function, the area under the standard normal pdf to the left of x.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990