**University of Illinois** Fall 2014

# ECE 313: Hour Exam I

Wednesday, October 8, 2014
7:00 p.m. — 8:15 p.m.
Section B in 151 Everitt Lab, Sections C & E in 141 Loomis Lab, Section D in 100 MSEB

1. **[14 points]** Suppose $n$ solar cells are in a linear array, in positions 1 through $n$, for some $n \geq 4$. Suppose exactly two of the cells fail, with all possible choices of which two being equally likely.

   (a) (6 points) What is the probability the two cells that fail are next to each other? Simplify your answer.

   (b) (8 points) Given that at least one of the two failures is among the first four cells in the array, what is the conditional probability that both failures were among the first four cells in the array? Simplify your answer.

2. **[10 points]** The lifetime of a machine $X$ in days is assumed to have a geometric distribution with *mean* $(1+\theta)$, where $\theta$ is an unknown positive real number.

   (a) (5 points) Find the maximum likelihood estimate $\hat{\theta}_{\mathrm{ML}}$ if $X$ is observed to equal 10.

   (b) (5 points) Now suppose the machine lifetime $X$ has the geometric distribution with *parameter* $p = 0.2$. Find the probability the machine will fail on day 12, given it has already lasted 10 days.

3. **[24 points]** A bag contains 4 orange and 6 blue t-shirts. Two t-shirts are chosen from the bag at random and moved into a second bag, which already contains one orange t-shirt. Then a t-shirt is chosen from the second bag at random.

   (a) (10 points) Let $X$ denote the number of blue t-shirts moved from the first bag to the second bag. Find the pmf of $X$.

   (b) (8 points) What is the probability the t-shirt chosen from the second bag is blue?

   (c) (6 points) What is the conditional probability $X = 2$, given the t-shirt drawn from the second bag is blue?

4. **[14 points]** The bit error probability of a communication system is estimated by using it to repeatedly transmit a single bit and seeing if the bit is received correctly or in error. Suppose 200,000 trials are used to estimate the probability of bit error.

   (a) (7 points) Find the (two-sided) width of the confidence interval for a 95% confidence level.

   (b) (7 points) How many trials are needed to shrink the confidence interval by a factor of 10 for the same level of confidence?

5. **[24 points]** Assume $X$ is a random variable describing observations under two possible hypotheses. Under hypothesis $H_0$, $X$ has a pmf given by $p_0(k) = 1/3$, for $k = 1,2,3$; under hypothesis $H_1$, $X$ has the pmf given by $p_1(1) = 1/2$ and $p_1(k) = 1/8$, for $k = 2,3$, and $p_1(k) = 1/4$, for $k = 4$.

   (a) (6 points) Describe the ML decision rule for this problem.

   (b) (6 points) Find $p_{\text{false alarm}}$ for the ML rule.

   (c) (6 points) Describe the MAP decision rule for this problem, assuming $\pi_0 = 2/3$ and $\pi_1 = 1/3$.

(d) (6 points) Does there exist a prior probability $\pi_1 > 0$ for which the MAP decision rule will always decide in favor of hypothesis $H_0$? Explain.

6. **[14 points]** You are taking two books with you for the Thanksgiving break. With probability 0.5, you will like the first book. With probability 0.4, you will like the second book.

   (a) (7 points) Find a strictly positive lower bound on the probability you will like neither book.

   (b) (7 points) Find the probability you will like neither book if, in addition to the assumptions above, you assume you will like both books with probability 0.3.