University of Illinois Fall 2013

ECE 313: Hour Exam I

Monday October 14, 2013
7:00 p.m. — 8:20 p.m.
Library 66, Sections X and C (9 a.m. and 10 a.m.)
Altgeld Hall 314, Sections D and E (11 a.m. and 1 p.m.)

Name: (in BLOCK CAPITALS) ____

University ID Number: __________________________

Signature: ____

Section: ☐ X, 9:00 a.m., ☐ C, 10:00 a.m. ☐ D, 11:00 a.m. ☐ E, 1:00 p.m.

Grading

1. 8 points ______________
2. 8 points ______________
3. 8 points ______________
4. 16 points ______________
5. 20 points ______________
6. 20 points ______________
7. 20 points ______________

Total (100 points) ____________

Instructions

This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, tablets, iPads, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of seven problems worth a total of 100 points. The problems are not weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK. Answers without appropriate justification will receive very little credit. If you need extra space, use the back of the previous page.

1. **[8 points]** What is $\mathsf{E}[X^2]$ for a Poisson random variable X with mean 6 ?

$\mathsf{E}[X^2] =$

2. **[8 points]** If Y is a geometric random variable with parameter $\frac{1}{2}$, what is $\mathsf{var}(3 - 2Y)$?

$\mathsf{var}(3 - 2Y) =$

3. **[8 points]** Let X denote the number of Heads that occur on 10 independent tosses of a coin with $P\{\text{Heads}\} = p$. Given that $m > 0$ heads were observed on the 10 tosses, what is the conditional probability that the second toss resulted in Heads?

$P\{\text{2nd toss is Heads} \mid X = m\} =$

4. **[16 points]** A biased coin is tossed repeatedly until a Head occurs for the first time. The probability of Heads in each toss is $\frac{1}{3}$. Let X denote the number of tosses required until the first Head shows.

   (a) **[8 points]** Find $E[X]$.

   $E[X] =$

   (b) **[8 points]** Find $P\{X = 7 \mid X > 5\}$.

   $P\{X = 7 \mid X > 5\} =$

5. **[20 points]** While debugging a software program, a student has narrowed it down to one of two bugs. Bug 1 leads to error message 1 with probability $\frac{1}{5}$ and error message 2 with probability $\frac{4}{5}$. Bug 2 leads to error message 1 with probability $\frac{2}{3}$ and error message 2 with probability $\frac{1}{3}$. The student also has the knowledge that the bug 1 is 5 times more likely to appear than bug 2, and that the two bugs cannot exist simultaneously.

   (a) **[10 points]** What is the probability that error message 1 appears?

$P\{\text{error message 1 appears}\} =$

   (b) **[10 points]** Given that error message 1 appears, what is the probability that the system has bug 1?

$P\{\text{system has bug 1} \mid \text{error message 1 appears}\} =$

6. **[20 points]** Consider events A, B, C, and D with probabilities $P(A) = 1/5$, $P(B) = 3/5$, $P(C) = 2/5$, and $P(D) = 3/5$, and suppose that $P(B \mid A) = 1/2$.

(a) **[7 points]** Find $P(A^c B)$ and $P(A \mid B)$.

$P(A^c B) =$

$P(A \mid B) =$

(b) **[5 points]** If A and C are independent, find $P(A^c C)$.

$P(A^c C) =$

(c) **[8 points]** If A and D are mutually exclusive, find as tight upper and lower bounds as possible on $P(BD)$.

$\leq P(BD) \leq$

7. **[20 points]** Consider a regular 8x8 chessboard, which consists of 64 squares in 8 rows and 8 columns.

   (a) **[10 points]** How many different rectangles, comprised entirely of chessboard squares, can be drawn on the chessboard? *Hint:* there are 9 horizontal and 9 vertical lines in the chessboard.

Number of rectangles =

   (b) **[10 points]** One of the rectangles you counted in part (a) is chosen at random. What is the probability that it is square shaped?

$P\{\text{square shaped rectangle}\} =$