ECE 313 (Section B)
In-Class Project 4 - Tuesday, October 28

Write the names and NetIDs of your group members here:

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Two biomedical signals, the blood pressure (ABP) and the heart rate (HR), are measured to detect the abnormalities of a patient in an intensive care unit (ICU). Assume that the blood pressure sensor outputs a value X and the heart rate sensor outputs a value Y. Both X and Y outputs have possible values of {0, 1, 2}, representing different ranges of ABP and HR values, with larger numbers tending to indicate that a patient abnormality is present.

Let H_0 be the hypothesis there is no abnormality, and H_1 be the hypothesis an abnormality is present. The likelihood matrices for X and for Y are shown:

	$X=0$	$X=1$	$X=2$
H_1	0.1	0.3	0.6
H_0	0.8	0.1	0.1

	$Y=0$	$Y=1$	$Y=2$
H_1	0.1	0.1	0.8
H_0	0.7	0.2	0.1

Suppose, given one of the hypotheses is true, the sensors provide conditionally independent readings, so that:

$$P(X=i, Y=j \mid H_k) = P(X=i \mid H_k).P(Y=j \mid H_k) \quad for\ i, j \in \{0,1,2\}\ and\ k \in \{0,1\}$$

a) Find the likelihood matrix for the observation (X, Y), and indicate the ML decision rule. To be definite, break ties in favor of H_1.

b) Find $p_{false-alarm}$ and p_{miss} for the ML rule found in part (a).

c) Suppose, based on past experience, prior probabilities are assigned as: $(p_0, p_1) = (0.8, 0.2)$. Compute the joint probability matrix and indicate the MAP decision rule.

d) For the MAP decision rule, compute $p_{false-alarm}$, p_{miss}, and the probability of error p_e. ($p_e = \pi_0 p_{false-alarm} + \pi_1 p_{miss}$)

e) Using the same priors as in part (c), compute the unconditional error probability for the ML rule from part (a). Is it smaller or larger than the p_e found for the MAP rule in part (d)?

Please note that the values of prior probabilities here are different from the values that we used in the class project.

Solution: (a) The likelihood matrix for observation (X, Y) is the following.

$(X,Y) \rightarrow$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
H_1	0.01	0.01	$\underline{0.08}$	0.03	$\underline{0.03}$	$\underline{0.24}$	0.06	$\underline{0.06}$	$\underline{0.48}$
H_0	$\underline{0.56}$	$\underline{0.16}$	0.08	$\underline{0.07}$	0.02	0.01	$\underline{0.07}$	0.02	0.01.

The ML decisions are indicated by the underlined elements. The larger number in each column is underlined, with the tie in case $(0, 2)$ broken in favor of H_1, as specified in the problem statement. Note that the row sums are both one.

(b) For the ML rule, $p_{\text{false\_alarm}}$ is the sum of the entries in the row for H_0 in the likelihood matrix that are not underlined. So $p_{\text{false\_alarm}} = 0.08 + 0.02 + 0.01 + 0.02 + 0.01 = 0.14$.
For the ML rule, p_{miss} is the sum of the entries in the row for H_1 in the likelihood matrix that are not underlined. So $p_{\text{miss}} = 0.01 + 0.01 + 0.03 + 0.06 = 0.11$.

(c) The joint probability matrix is given by

$(X,Y) \rightarrow$	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(2,0)$	$(2,1)$	$(2,2)$
H_1	0.002	0.002	0.016	0.006	0.006	$\underline{0.048}$	0.012	0.012	$\underline{0.096}$
H_0	$\underline{0.448}$	$\underline{0.128}$	$\underline{0.064}$	$\underline{0.056}$	$\underline{0.016}$	0.008	$\underline{0.056}$	$\underline{0.016}$	0.008.

(The matrix specifies $P(X = i, Y = j, H_k)$ for each hypothesis H_k and for each possible observation value (i, j). The 18 numbers in the matrix sum to one. The MAP decisions are indicated by the underlined elements in the joint probability matrix. The larger number in each column is underlined.)

(d) For the MAP rule,

$$p_{\text{false\_alarm}} = P\left[(X, Y) \in \{(1, 2), (2, 2)\} | H_0\right] = 0.01 + 0.01 = 0.02,$$

and

$$p_{\text{miss}} = P((X, Y) \notin \{(1, 2), (2, 2)\} | H_1) = 1 - P((X, Y) \in \{(1, 2), (2, 2)\} | H_1) = 1 - 0.24 - 0.48 = 0.28.$$

Thus, for the MAP rule, $p_e = (0.8)(0.02) + (0.2)(0.28) = 0.072$. (This p_e is also the sum of the probabilities in the joint probability matrix that are not underlined.)

(e) Using the conditional probabilities found in (a) and the given values of π_0 and π_1 yields that for the ML rule: $p_e = (0.8)(0.14) + (0.2)(0.11) = 0.134$, which is larger than the value 0.072 for the MAP rule, as expected because of the optimality of the MAP rule for the given priors.
